ABSTRACT. The shear lag method is developed and applied to investigate the piezoelectric response of a bi-material structure under static loading with the influence of a moisture and temperature field and with a possible delamination along the interface. The investigation starts with a bi-material structure for which the first piezoelectric plate is pre-cracked, and the second plate is intact and purely elastic. It is also assumed that the properties of the materials are linear and the changes of strain, temperature, and moisture in the structure will be considered within the constant and linear range.

KEY WORDS: Shear-lag hygrothermoelastic model, bi-material structure

1. Introduction

A solid under the coupled effects of mechanical, electrical, thermal, and moisture fields is considered as a hygrothermoelastic medium. Adaptive composite structures, composed of piezoelectric material(s) and exposed to environmental effects are examples of such a medium. These effects on laminated composite plates have received a remarkable amount of recent attention [1-4]. However, the effects of these four fields together have never before been studied simultaneously.

In analyzing laminated elastic plates, many theories have been developed. Equivalent-single-layer (ESL) theories in two dimensions, such as classical laminate theory (CLT) and first- and higher-order shear deformation theories [5]. Piezoelectric and thermopiezoelectric effects were incorporated to yield the exact solutions for the laminates composed of these materials [3, 6, 7]. A layerwise theory first appeared in
J. Ivanova, V. Valeva, T. Petrova, W. Becker

Later, discrete-layer models (DLM), using the layerwise theories, were developed for analyzing laminated piezoelectric plates [9-10]. However, none of these studies has considered the four fields combined here.

The main goal is to model the behavior of hygrothermopiezoelectric composite structures and to obtain the system of ordinary differential equations describing the piezo-electrical, thermal and moisture effects on the 1-D response of a bi-material structure. The closed form of the final is applied to the example of the material structure under given load and physical characteristics. The obtained numerical results are illustrated on figures and discussed.

2. Statement of the problem and solution

The shear lag analysis will be applied to the model of the unit bi-material cell according to Fig. 1:

Fig. 1 Model of the unit cell of a bi-material structure

where $2h_A, 2l, l$, $T, H$ ($\kappa = A, B$) are the thicknesses of plates $A$ and $B$, the length of the unit cell, the debond length, the temperature change and the moisture concentration change, respectively. The external mechanical load is applied as $\varepsilon_0$, while the electric field is given by the electric displacement $D_0$.

The first plate $A$ is assumed to be transversal isotropic elastic with piezoelectric properties and sensitive to thermal effects, while the second plate $B$ is isotropic, sensitive to thermal and moisture effects. Both plates are connected with a zero thickness isotropic elastic material line (interface $I$), working only on shear, while both plates work on extension. According to the shear lag hypothesis the 1-D system of differential equations for the equilibrium of the unit cell is obtained from the 3-D case, i.e.:

$$
\begin{align*}
\frac{d\sigma_A}{dx} - \frac{\tau_1}{2h_A} &= 0, \\
\frac{d\sigma_B}{dx} + \frac{\tau_1}{2h_B} &= 0, \\
\frac{dD_A}{dz} &= 0, \\
\frac{d^2T}{dx^2} &= 0.
\end{align*}
$$

Boundary and contact conditions are:

$$
\begin{align*}
\varepsilon_A(l) &= \varepsilon_0, \\
\varepsilon_B(l) &= \varepsilon_0, \\
u_A(0) &= 0, \\
\sigma_A(0) &= 0, \\
T(0) &= T_0, \\
T(l) &= T_1, \\
H(0) &= H_0, \\
H(l) &= H_1.
\end{align*}
$$
where $\sigma_{\kappa}, \epsilon_{\kappa} \ (\kappa = A,B)$ are the stresses and strains, $\tau_{i}$ is the interfacial shear stress, $D_{\Theta}$ is the electric displacement of plate A. The solution of the third equation from (2.1) is $D_{\Theta} = D_{0}, \ 0 \leq x \leq l$.

The following kinematical assumptions hold: $\epsilon^{\text{tot}} = du/dx$ and $\epsilon^{\text{moh}} = \epsilon^{\text{tot}} - \epsilon^{\text{pol}} - \epsilon^{\text{n}}$. The temperature and moisture have the form: $T = T_{0} - (T_{1} - T_{0})(1-x/l), \ H = H_{0} - (H_{1} - H_{0})(1-x/l)$. According to the considered materials of plates A and B, for plate A we have $\epsilon^{n} = 0$, for plate B $\epsilon^{\text{pol}} = 0$.

The constitutive equations for the plates A and B as well as for the interface I are as follows:

$$E_{\Theta} = \frac{D_{\Theta}}{\epsilon_{33}} - \frac{\rho_i}{\epsilon_{33}} \left[ T_{0} - (T_{1} - T_{0})(1-x/l) \right] - \frac{\epsilon_{i1}^{*}}{\epsilon_{33}} \frac{du}{dx} \ , \ \tau_{i} = G_{i} \frac{u_{A} - u_{B}}{h_{A} + h_{B}}$$

$$\sigma_{A} = \left[ c_{11}^{*} + \frac{\epsilon_{31}^{*}}{\epsilon_{33}} \frac{du}{dx} - \frac{\epsilon_{i1}^{*}}{\epsilon_{33}} D_{\Theta} \right] \left[ T_{0} - (T_{1} - T_{0})(1-x/l) \right]$$

$$\sigma_{B} = E_{B} \frac{du}{dx} - E_{B} \alpha_{B} \left[ T_{0} - (T_{1} - T_{0})(1-x/l) \right] - E_{B} \beta_{B} \left[ H_{0} - (H_{1} - H_{0})(1-x/l) \right]$$

$$c_{11}^{*} = c_{11} - \frac{c_{13}^{2} c_{33} - 2 c_{12} c_{13}^{2} + c_{13}^{3}}{c_{13}^{2} - c_{12}} , \ \ \ \epsilon_{31}^{*} = \epsilon_{31} - \frac{\epsilon_{31}^{*}}{\epsilon_{33}} \left( c_{11} c_{33} - c_{13}^{2} + c_{13} \epsilon_{33} \right) c_{13} - c_{12}$$

$$\epsilon_{33}^{*} = \epsilon_{33} - \frac{c_{12}^{2} - 2 c_{12} c_{13}^{2} + c_{13}^{3}}{c_{13}^{2} - c_{12}} , \ \ \ \alpha_{11}^{*} = \alpha_{11} - \frac{\alpha_{11} c_{13}^{2} - c_{13}}{c_{13} c_{33} - c_{12}}$$

$$p_{i}^{*} = p_{i} + \frac{\alpha_{11} c_{13} - c_{12} \epsilon_{33}}{c_{13}^{2} - c_{12}}$$

and $\sigma_{\kappa}$ and $u_{\kappa}, \ (\kappa = A,B)$ are stresses and displacements and $E_{\Theta}$ is the electric gradient for plate A; $c_{ij}, \epsilon_{ij}, \alpha_{ij}, \beta_{ij}, \ (i, j = 1, 2, 3)$ are elastic constants (measured at constant electric field), piezoelectric and dielectric constants (measured at constant strain), thermal stress coefficients, pyroelectric coefficient for plate A; $E_{B}, \alpha_{B}, \beta_{B}$ are Young’s modulus, thermal and moisture expansion coefficients for plate B and $G_{i}$ is the shear modulus of the interface, respectively. The total hygrothermalpiezoelectric stresses and displacements for the model (see Fig. 1) read:

$$u_{A} = \frac{1}{\lambda} \left( \epsilon_{0} - \frac{R}{\lambda^{2}} l \right) \left[ \frac{ch\lambda(l-x)}{sh(\lambda l)} + \left( \frac{\lambda l}{\xi} - 1 \right) \frac{ch(\lambda l)}{sh(\lambda l)} + (\lambda x) \right] - \frac{1}{\xi} \left( P - \frac{R}{\lambda^{2}} \right) + \frac{R}{\lambda^{2}} \frac{x^{2}}{2}$$

$$u_{B} = \frac{1}{\lambda} \left( \epsilon_{0} - \frac{R}{\lambda^{2}} l \right) \left[ \left( 1 - \frac{\lambda^{2}}{\xi} \right) \frac{ch\lambda(l-x) - ch(\lambda l)}{sh(\lambda l)} + (\lambda x) \right] + \frac{R}{\lambda^{2}} \frac{x^{2}}{2}$$

$$\tau_{i} = \frac{G_{i}}{(h_{A} + h_{B})} \left( \epsilon_{0} - \frac{R}{\lambda^{2}} l \right) \left( \frac{ch\lambda(l-x)}{sh(\lambda l)} + \frac{R}{sh(\lambda l)} - P \right)$$

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The length of an interfacial debonding is found from the condition that the interface shear stress reaches its failure limit \( \tau_{cr} \), i.e.

\[
\sigma_A = \left( e_{11}^* + e_{33}^* \right) \left( e_0 - \frac{R}{\lambda^2} \left[ 1 - \frac{\sinh[\lambda(l - x)]}{\sinh(\lambda l)} \right] + \frac{R}{\lambda^2} x \right) - \frac{e_{33}^*}{e_{33}^*} D_0 - \\
\left( \alpha_{11}^* - \frac{e_{33}^*}{e_{33}^*} p_{33}^* \right) T_i - (T_i - T_0)(1 - \frac{x}{l})
\]

\[
\sigma_B = E_B \left( e_0 - \frac{R}{\lambda^2} \left[ 1 - \left( 1 - \frac{\lambda^2}{\xi} \right) \frac{\sinh[\lambda(l - x)]}{\sinh(\lambda l)} \right] + \frac{R}{\lambda^2} x \right) - \\
E_B \beta_\alpha [H_i - (H_i - H_0)(1 - \frac{x}{l})]
\]

\[
E_i = \frac{D_0}{e_{33}^*} - \frac{e_{33}^*}{e_{33}^*} \left( e_0 - \frac{R}{\lambda^2} \left[ 1 - \frac{\sinh[\lambda(l - x)]}{\sinh(\lambda l)} \right] + \frac{R}{\lambda^2} x \right) - \frac{p_{33}^*}{e_{33}^*} [T_i - (T_i - T_0)(1 - \frac{x}{l})]
\]

The length of an interfacial debonding is found from the condition that the interface shear stress reaches its failure limit \( \tau'' \), i.e. \( \tau_i(l) = \tau'' \), i.e.

\[
l_\tau = \frac{1}{\lambda} \ln \left( C \pm \sqrt{C^2 - 1} \right), \quad C = \frac{\sinh(\lambda l)}{\lambda \left( e_0 - \frac{R}{\lambda^2} \right)} \left[ \frac{(h_a + h_b) \xi}{G_i} \tau'' - \frac{R}{\lambda^2} + P \right]
\]

3. Numerical example and results

The geometry, the mechanical and electric loads, the temperatures and moisture concentrations are given as follows: \( l = 140 \text{ mm} \), \( h_a = 2 \text{ mm} \), \( h_b = 5 \text{ mm} \), \( e_0 = 0.005 \pm 0.02 \), \( D_0 = 0.5 \text{ C/m}^2 \), \( T_0 = 200 \text{ K} \), \( T_i = 370 \text{ K} \), \( H_0 = 0.5 \text{ (\%) } \), \( H_i = 2.5 \text{ (\%) } \). The interface is made from polyacrylate glue and is given with \( G_i = 800 \text{ MPa} \) and \( \tau'' = 18 \text{ MPa} \). The materials of the plates \( A \) (PZT-5H [11]) and \( B \) (Carbon fiber/epoxy – isotropic in \( x \)-direction [12]) have the following properties: elastic constants (\( GPa \)): \( c_{11} = 126 \), \( c_{12} = 55 \), \( c_{33} = 53 \), \( c_{13} = 117 \), \( E_B = 10.3 \); piezoelectric constants (\( C/m^2 \)): \( e_{33} = -6.5 \), \( e_{33} = 23.3 \); dielectric constant (\( C/Vm \)): \( e_{33} = 1.3 \times 10^{-8} \); pyroelectric coefficient (\( C/Km^2 \)): \( p_{33} = -5.4831 \times 10^{-6} \); thermal stress coefficient (\( N/m^2K \)): \( \alpha_{11} = 1.97382 \times 10^6 \), \( \alpha_{33} = 1.4165 \times 10^6 \); thermal \( \alpha_B = 22.5 \times 10^{-6} \text{ (l/K) } \) and moisture \( \beta_B = 0.006 \text{ (l/\% )} \) expansion coefficients. The following cases for material and physical characteristics of the plates \( A \) and \( B \) are considered:

**Case 1:** plate \( A \): piezo-elastic, plate \( B \): elastic

**Case 2:** plate \( A \): piezo-thermo-elastic, plate \( B \): hygro-thermo-elastic
Fig. 2 describes the behaviour of the interface shear stress and electric gradient along the length of the bi-material unit cell \((0 \leq x \leq l)\) at \(\varepsilon_0 = 0.01\). It can be seen, that the presence of thermal and moisture excitation also leads to a reduction of the interface shear stress and electric gradient values.

Fig. 2. Interface shear stress and electric gradient as a function of \(x\) \((0 \leq x \leq l)\)

![Diagram](image1)

Fig. 3. (a) Behavior of the debond length for different values of the mechanical load; (b) Electric gradient as a function of the debond length

The debond length (Fig. 3a) grows up with increasing the mechanical load \(\varepsilon_0 = 0.005 \pm 0.02\). The presence of the temperature and the moisture decreases the values of the debond length. It can be observed that for some values of the mechanical load \(\varepsilon_0 > 0.01\) the debond length has higher values than for the case 1. The indirect dependence of the electric gradient from debond length is shown in Fig. 3b. It is evident that to the value of the debond length \(l_e\), the respective value of electric gradient \(E_{zd}\) uniquely corresponds and vice versa. The last conclusion could be proposed as a possible criterion for detecting the interface debond length.
4. Conclusion

The modelling of the idealized straight line part of a bi-material structure for industrial application by the analytical and simple shear lag model is done. The analysis and detection of the possible interface delamination through the change of the voltage, temperature and moisture for the hygrothermal piezoelectric bi-material structure at steady state behaviour is provided. The numerical example shows the strong influence of the temperature and of moisture, which is very important for the possible degradation of the interface of the bi-material structure.

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