ABSTRACT. The performance of buried steel arch system being subjected to seismic waves is considered and studied in the paper. Numerical 2D model of elastic layered half space is accomplished by direct boundary element method while the steel arch is modeled by frame finite elements. Connection between boundary and finite elements implies that both types of elements have common displacements and equal nodal forces. The mechanism of frame loading is carried out through seismic wave propagation and wave passage. The model allows for analyzing the plane strain state in time domain. Linear elastic behavior of the soil and steel arch is assumed. Seismic input is considered to be elastic waves excited and propagating in the supporting half-space. Analysis methodology is based on Fourier analysis of the soil-arch interacting system. This approach is suitable to be applied for performance-based seismic design of culvert and tunnel systems as underground structures.

KEY WORDS: Soil-Structure Interaction, Wave Propagation, Boundary Element Method.

1. Introduction
In the current study we consider steel culvert structure partially or entirely buried in nonhomogeneous soil deposit. The culvert is sufficiently long in the direction orthogonal to its cross sectional plane and as such it is considered subjected to plane-strain state. Seismic type of excitation is adopted for the model, where the excitation zone is assumed to be the supporting half-space.

2. Bearing structure data
The cross section of the bearing structure is considered to be semicircle with the dimensions and geometry shown in Figure 2.1 a).

Figure 2.1. Bearing structure: a) scheme of the cross section, b) FEM discretization of the arch, c) BEM discretization of the footing.
The culvert bearing structure is steel arch made of sheet iron shells, with the geometrical data given in Table 2.1. The numerical model of such a thin structure is impossible to be comprehensively modeled at a macro level. That’s why the numerical model of the thin steel structure is composed of finite frame elements with cross section, geometrical and material data given in Table 2.1.

Table 2.1. Sheet iron profile, geometrical and material data.

<table>
<thead>
<tr>
<th>t</th>
<th>A</th>
<th>Aq</th>
<th>I</th>
<th>E</th>
<th>G</th>
<th>ν</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5×10⁻⁴</td>
<td>6.97×10⁻⁴</td>
<td>5.81×10⁻⁴</td>
<td>1.72×10⁻⁵</td>
<td>2.1×10⁶</td>
<td>8.1×10⁷</td>
<td>0.3</td>
<td>0</td>
</tr>
</tbody>
</table>

The foundation of the structure is considered to be of two independent concrete footings with material properties shown in Table 2.2. The bearing structure is assumed to be hinge supported over these two independent concrete strip footings, coupled through the soil.

Table 2.2. Material properties of the concrete strip footings.

<table>
<thead>
<tr>
<th>E [kN/m²]</th>
<th>G [kN/m²]</th>
<th>ν</th>
<th>ρ [t/m³]</th>
<th>c_s [m/s]</th>
<th>c_p [m/s]</th>
<th>ζ</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5×10⁷</td>
<td>1.0×10⁷</td>
<td>0.25</td>
<td>2.5</td>
<td>2000</td>
<td>3464</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The finite element method is employed in modeling and dynamic analysis of the bearing structure. The body of the structure is modeled with finite frame elements (Bernoulli beam elements) which axes are located at the centroid of the structure. The arch of the structure is divided into 80 frame elements with equal lengths. Each one of the supporting strip footings is most appropriate to be discretized with plane finite element mesh. Nevertheless for convenience in the current study the strip footings are discretized with boundary element mesh as shown in Figure 2.1 c). The boundary element mesh for one strip footing consists of totally 28 boundary elements delineating the contour of the footing. Dynamic stiffness of the structure is obtained from the well-known stiffness formulation of the finite element method (FEM) in frequency domain, i.e. the following expression:

\[ [S] = [K] + i\omega [C] - \omega^2 [M], \]

where \([K]\), \([C]\) and \([M]\) are the structural stiffness, viscous damping and mass matrices respectively.

3. Soil deposit data

The soil stratum, studied here, consists of two layers entirely embedded in a flexible half-space, as shown in Figure 3.1. The culvert structure is considered to be situated at the boundary between the two upper layers. Different layer IDs and their material properties are present in Table 3.1:

Table 3.1. Material properties of the soil deposit.

<table>
<thead>
<tr>
<th>Layer ID</th>
<th>E [kN/m²]</th>
<th>G [kN/m²]</th>
<th>ν</th>
<th>ρ [t/m³]</th>
<th>c_s [m/s]</th>
<th>c_p [m/s]</th>
<th>ζ</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80000</td>
<td>33333,33</td>
<td>0.2</td>
<td>2.4</td>
<td>117.85</td>
<td>192.45</td>
<td>0.05</td>
<td>Gravel Bed</td>
</tr>
<tr>
<td>2</td>
<td>35000</td>
<td>15217.39</td>
<td>0.15</td>
<td>1.9</td>
<td>89.494</td>
<td>139.47</td>
<td>0.05</td>
<td>Layer</td>
</tr>
<tr>
<td>3</td>
<td>15000</td>
<td>5769.23</td>
<td>0.3</td>
<td>1.8</td>
<td>56.614</td>
<td>105.91</td>
<td>0.05</td>
<td>Layer</td>
</tr>
<tr>
<td>4</td>
<td>47500</td>
<td>18554.70</td>
<td>0.28</td>
<td>2.0</td>
<td>96.32</td>
<td>174.25</td>
<td>0.05</td>
<td>Half-Space</td>
</tr>
</tbody>
</table>
According to the location of the keystone of the circular arch we consider two cases: 1) the arch keystone is buried below the soil free surface: this stage corresponds to the operational state of the culvert, where the surface of the most top soil layer is covered up with gravel bed of thickness 0.2 meters in depth; 2) the arch keystone is at certain level above the soil free surface: this stage corresponds to the backfilling state of the culvert, during its construction. It is assumed that the backfilling material is the same as the top soil layer material. According to the two states of the arch (fully or partially buried in the most top soil layer) we consider two different shapes for the surficial layer: one with the backfilling area included and another without it, as shown in Figure 3.1 a).

The hybrid boundary-finite element method is employed in modeling and dynamic analysis of the soil stratum. The infinite soil domain is modeled with linear boundary elements and afterward is converted into several macro-finite elements, shown as separate domains in Figure 3.1 b). Brief information for the discretization of each of the domain boundaries is given in the corresponding figure in framed numbers. Herein the boundary elements are generated and assembled together to represent the numerical model of the soil deposit. It is obvious that this boundary element model could be directly used for the evaluation of the behavior of the soil stratum. Nevertheless since we need to couple the soil boundary element model with the structure finite element model we have to transform each boundary element domain into a single macro-finite element that could be easily assembled with any other group of finite elements. For this purpose the system matrices of the boundary element method (BEM) for the j-th particular soil domain are used in the generation of the dynamic stiffness matrix for each one of the boundary elements (superscript “e”) delineating the boundary of this domain, i.e:

\[
[S] = [L]^T [G_j^{-1} H_j] \]

where \([G_j]\) and \([H_j]\) are the BEM system matrices for the j-th boundary element domain satisfying the equation \([H_j] \{ \ddot{u}_j \} = [G_j] \{ \ddot{p}_j \}\) and the matrix
\[ [T] = \int_{e} N^T \cdot N \, ds \] is the one we need to transform element tractions into nodal forces. Herein the element stiffness matrices are assembled together to obtain the frequency dependent dynamic stiffness matrix \( [S(\omega)] = \int_{e} [S(\omega)] \, ds \) representing the \( j \)-th soil domain. At last all the macro-finite elements are assembled together to obtain the dynamic stiffness matrix for the whole soil stratum.

4. Soil-structure coupling
Coupling of the soil and the structure model is accomplished identically to the coupling of two finite element regions in FEM. The terms of the two stiffness matrices for both models (the soil and the structure model) corresponding to the coincident nodes at the contact boundaries between these two models are simply added together to obtain the coupled stiffness matrix for the entire model. This coupling procedure satisfies equilibrium and compatibility conditions expressed in equal and opposite nodal forces and equal nodal displacements at the boundary between the soil and structure model. Coupling of the soil and structure systems could be symbolically expressed as:

\[
\begin{bmatrix}
S_{ss}^{\text{FEM}} & S_{sc}^{\text{FEM}} \\
S_{cs}^{\text{FEM}} & S_{cc}^{\text{FEM}} + S_{cg}^{\text{BEM}} \\
0 & S_{cg}^{\text{BEM}} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
u_s \\
u_c \\
u_g
\end{bmatrix} =
\begin{bmatrix}
F_s \\
F_c \\
F_g
\end{bmatrix},
\]

where the subscript symbol denotes: \( s \) – structure, \( c \) – contact zone (the soil-structure intersection domain), \( g \) – ground (the soil strata).

5. Excitation data
In this case evenly distributed P or SV body waves propagate in the supporting half-space at a certain angle of incidence to the top surface. The simulation of the body wave incidence in the half-space is accomplished according to the substructure formulation exposed in [3]. The input into the system is taken to be the free field motion of the supporting half-space (in terms of free surface) resulting from the propagation of evenly distributed body waves. As incident wave displacement time-history function is chosen the harmonic function \( u_w(t) = \tilde{u}_w \sin(\omega t) \), where:

\( \tilde{u}_w = 0.1 \, [\text{m}] \) is the wave amplitude and \( \omega = 4\pi \, [\text{rad/s}] \) is the circular frequency of the wave’s oscillations.

6. Analysis procedure
The dynamic behavior of the soil, the structure and the soil-structure interface is evaluated through Fourier analysis. This type of analysis constitutes of the following three stages: 1) Fourier transformation of the system of governing equations and the input; 2) Subjecting the frequency dependent system to the input and performing analysis in frequency domain; 3) Inverse Fourier transformation of the output.
7. Verification example

As verification model is used the half-space domain distorted from internal cavity in the shape of full circle and subjected to incident SV-waves, as shown in Figure 7.1. The material properties for the present half-space domain are given in Table 7.1. The excitation is uniformly distributed incident SV-waves inclined at certain angle $\theta_{inc}$ to the free surface. The time-history variation of the wave magnitude is considered to be the complex harmonic function $u_w(t) = \tilde{u}_w \cdot \exp(i \omega t)$, described by unit real amplitude $\tilde{u}_w = 1$[m] and excitation frequency $\omega = \pi$[rad/s].

![Figure 7.1](image1)

Figure 7.1. Numerical model of half-space domain distorted from internal inclusion in the shape of full circle and subjected to incident P- and SV-waves.

<table>
<thead>
<tr>
<th>Layer</th>
<th>E [kN/m$^2$]</th>
<th>G [kN/m$^2$]</th>
<th>v</th>
<th>$\rho$ [t/m$^3$]</th>
<th>$c_s$ [m/s]</th>
<th>$c_p$ [m/s]</th>
<th>$\zeta$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.666(6)</td>
<td>1</td>
<td>0.333(3)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>Half-Space</td>
</tr>
</tbody>
</table>

The plots presented in Figure 7.2 illustrate the distributions of the horizontal and vertical displacement amplitude components along the free surface of the considered media, subjected to incident SV-waves at angle $\theta_{inc} = 10^\circ$. These graphs conclude that the BEM-FEM results, derived by the present numerical modeling, are in very good agreement with the results given from Yu & Dravinsky in [4].

![Figure 7.2](image2)

Figure 7.2. Distribution of the horizontal $|\tilde{u}_x|$ and vertical $|\tilde{u}_z|$ displacement amplitude components along the free surface of the half-space, subjected to incident SV-waves at angle $\theta_{inc} = 10^\circ$. 
8. Experimental results
All the necessary information for the results presented here could be found in Chapter 3 and Figure 3.1. The applied excitation is incident from the supporting half-space body waves at a certain angle $\alpha$ to the free surface. The excitation data is given in Section 5. It is the purpose of this study to investigate the influence of the incident seismic body waves on the behavior of the embedded in the soil deposit culvert structure. The plots presented in Figure 8.1 depict the distributions of the horizontal and vertical displacement components along the free surface in the moment of time $t = 0.22 \text{ [s]}$.

![Figure 8.1](image_url)

Figure 8.1. Distribution of the horizontal ($u_H$) and vertical ($u_V$) displacement components along the free surface in the moment of time $t = 0.22 \text{ [s]}$ for the case of incident P-wave.

9. Conclusions
In the current work a comprehensive study is performed for evaluation of the influence of the dynamic soil-structure interaction phenomena on the behavior of a culvert structure entirely embedded in nonhomogeneous soil deposit. The soil-culvert system is subjected to seismic body P- or SV-waves, incident in the soil deposit. The seismic body wave incidence is implicitly modeled following the substructure approach, where the excitation is transformed into equivalent nodal forces, applied in the nodes of the model and determined on the base of the free field motions in the supporting half-space, subjected to the same body wave incidence. The study performed here clearly demonstrates the influence of the seismic body waves on the embedded culvert structure.

REFERENCES