APPLICATION OF GEARING PRIMITIVES TO SKEW–AXES GEAR SET SYNTHESIS
PART 2: MATHEMATICAL MODEL FOR SYNTHESIS AND APPLICATIONS

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ABSTRACT. Spatial skew-axes gears are used to transform rotation between shafts with non-parallel and non-intersecting axes. Principles of mathematical modelling of a tooth contact synthesis for this general gearing case are discussed in this paper. The research shows that synthesis of the gear drives can be realized by application of a pitch contact point mathematical model. The approach requires application of gearing primitives (pitch configurations) to the synthesis of Spiroid and Helicon gear sets. These gears are alternative of a bevel gear with straight teeth that is incorporated in the driving of the robot’s hand, constructed in Mechanical Engineering Department at Gifu University, Japan.

KEY WORDS: Mathematical model, synthesis, pitch configurations

1. Introduction
The study is an extension of what is started in Part 1. The aim is to find out a solution of one of the basic tasks of the synthesis of skew-axes gears - synthesis of pitch configurations (pitch circles and pitch surfaces), which are primitives of their synthesis. The mathematical model and algorithms, that contains geometric
characteristics of pitch configurations, are illustrated in Fig. 2 of Part 1 of the current study. In Fig. 2 is shown only one alternative of pitch configurations - pitch circles.

2. Synthesis using a pitch contact point

The mathematical model for synthesis based on a pitch contact point ensures solving of two basic problems:

- synthesis of the pitch configurations;
- synthesis of the active tooth surfaces.

Pitch configurations: essence and definition. The present paper treats the kinematical theory of the spatial transformation of rotations in the context of defining the kinematic-geometric essence of basic elements of the mathematical models for synthesis of hyperboloid gears and their role in solving the gears synthesis. These elements have been called pitch configurations (pitch circles and pitch surfaces) and treat one actual but still disputable (as a content and terminology) part of the meshing theory.

From the performed analysis of the publications [1] and the comments above we can conclude that:

- If the law of transformation of rotations \( \omega_1 / \omega_2 = \text{constant} \) (\( \omega_i \) - angular velocity of the rotating link \( i \)) between fixed and crossed axes \( 1-1 \) and \( 2-2 \) (the shortest distance between them is \( a_w = \text{constant} \) and the angle between them is \( \delta = \angle(\overrightarrow{a_1}, \overrightarrow{a_2}) = \text{constant} \)) is given, and if the position of a point \( P \) (treated as a point of contact of conjugate tooth surfaces \( \Sigma_1 \) and \( \Sigma_2 \)) in the fixed space is known, then the diameters and the mutual position of the circles \( H_i \) \((i = 1, 2) \) are completely and uniquely determined. The circumferential velocity vectors \( \mathbf{V}_i \) \((i = 1, 2) \) of the common point \( P \), and the relative velocity vector \( \mathbf{V}_{12} \) at the same point, (i.e. the plane \( T_m \) where the coplanar vectors \( \mathbf{V}_i \) \((i = 1, 2) \) and \( \mathbf{V}_{12} \) lie), as well as the normal \( m-m \) to \( T_m \) at the point \( P \) are determined in a unique way, too.

- It is sufficient to know the mutual position of the crossed axes of rotation \( 1-1 \) and \( 2-2 \), and the position of the point \( P \) (as a common point of the tooth surfaces \( \Sigma_1 \) and \( \Sigma_2 \)) in the fixed space, in order the circles \( H_i \) \((i = 1, 2) \) to be completely and uniquely determined (as diameters and mutual position). The plane \( T_m \) formed by the tangents to the circles \( H_i \) \((i = 1, 2) \) at the point \( P \), as the normal \( m-m \) to \( T_m \) at the point \( P \) are uniquely determined, too.

Mathematical model for synthesis of pitch configurations. Let two crossed axes \( 1-1 \) and \( 2-2 \) representing the axes of rotations of the movable links of a three-link tooth mechanism be given in the fixed space. Their mutual position is defined by the angle \( \delta = \text{constant} \) (the angle between the angular velocity vectors \( \omega_1 \) and \( \omega_2 \) of the movable links \((i = 1, 2) \)) and the shortest distance \( a_w = \text{constant} \). The concrete study is performed when \( \delta \in (0, \pi) \). Each pitch circle \( H_i \) lies in a plane perpendicular to
the axis of rotation \( i-i \) of the movable link \( i \) and has a radius equal to the distance from the point \( P \) to the axis \( i-i \). The pitch plane \( T_m \) is determined by the tangents to the circles \( H_1^c \) and \( H_2^c \) at the pitch contact point \( P \). Besides, the pitch normal \( m-m \) to \( T_m \) is determined at \( P \). The study performs by means of the notations and the coordinate frames \( S_i(O_1, x_1, y_1, z_1) \) and \( S_2(O_2, x_2, y_2, z_2) \), introduced in Fig. 2 (Part 1). The dimensions and the mutual position of \( H_1^c \) and \( H_2^c \) are completely determined by the cylindrical coordinates \( a_i, r_i, \theta_i \) \((i=1, 2)\) of the contact point \( P \) in the systems \( S_i \) \((i=1, 2)\) and by the angles \( \delta_i \) \((i=1, 2)\) between the planes of \( H_i^c \) \((i=1, 2)\) and the normal \( m-m \).

The graphical model, shown in Part 1, is oriented to the synthesis of pitch circles. This type of pitch configurations are character for the traditional constructive types of spatial gears (hypoid ones, Spiroid and Helicon ones, etc.). We represent the radius-vector \( \overrightarrow{OP} \) and the unit vector \( m \) of the normal \( m-m \) by \( a_1, r_1, \theta_1 \) and \( a_2, r_2, \theta_2 \) and using \( a_w \) and \( \delta = 90^\circ \). Thus we get the following set of equations:

\[
\begin{align*}
\begin{aligned}
    r_1 \cos \theta_1 &= a_2, \\
    r_1 \sin \theta_1 &= a_w - r_2 \sin \theta_2, \\
    a_1 &= r_2 \cos \theta_2, \\
    \cos \delta_i \sin \theta_1 &= \cos \delta_2 \sin \theta_2, \\
    \sin \delta_1 &= \cos \delta_2 \cos \theta_2, \\
    \cos \delta_1 \cos \theta_1 &= \sin \delta_2.
\end{aligned}
\end{align*}
\]  

(2.1)

Later, the study of the system (2.1) will be performed taking into account the following geometric conditions \( \theta_1 \in \left[ 0, \frac{\pi}{2} \right] \), \( \theta_2 \in \left( 0, \frac{\pi}{2} \right) \), \( \delta_1 \in \left[ 0, \frac{\pi}{2} \right] \), \( \delta_2 \in \left[ 0, \frac{\pi}{2} \right] \). Each of the last three equations in (2.1) is a consequence of the other two, i.e. (2.1) is a set of 5 independent equations with 9 unknowns: \( a_w, \delta_1, r_1, a_1, \theta_1, \delta_2, r_2, a_2, \theta_2 \). Therefore, each solution of (2.1) is a function of 4 of them (we will consider them as free ones). We suppose that the independent (free) parameters are \( a_w, \delta_1, r_1 \) and \( a_1 \). We will look for analytical relations that they have to be fulfilled so that the set (2.1) to have a solution.

**Synthesis of orthogonal contacting pitch configurations.** Let us pay attention only to the cases oriented to the synthesis of Spiroid and Helicon gears, but also to such essential for the practice cases of orthogonal hyperboloid gears.

(a). \( \delta_1 = 0 \) and \( a_1 \neq 0 \).

This case refers to the orthogonal hyperboloid gears with external meshing, when the coaxial surfaces of the movable link \( i=i \) - reference, root and tip ones are of cylindrical form.
System (2.1) has the following unique solution

\[
\begin{align*}
\theta_1 &= 0, \quad \tan \theta_2 = \frac{a_w}{a_1}, \quad a_2 = r_1, \\
& \\
r_2 &= \frac{a_w}{\sin \theta_2}, \quad \delta_2 = \frac{\pi}{2},
\end{align*}
\]

in an arbitrary choice of \( a_w, \ r_1 \) and \( a_1 \). The parameters in (2.2) define the dimensions and the position of the pitch configurations corresponding to spatial high reduction gears of type Helicon (Fig. 1) [1, 2].

(b). \( \delta_1 > 0 \) and \( a_1 \neq 0 \)

In this case the condition for an existence of pitch configurations is

\[
a_1 \geq (a_w - r_1) \tan \delta_1,
\]

and the solution of the system is:

\[
\theta_1 = 0, \quad \tan \theta_2 = \frac{a_w}{a_1}, \quad a_2 = r_1,
\]

\[
r_2 = \frac{a_w}{\sin \theta_2}, \quad \delta_2 = \frac{\pi}{2},
\]

\[
\begin{align*}
\cot \theta_2 &= \frac{r_1 \tan \delta_1 + a_1}{a_w}, & \sin \theta_1 &= \frac{a_w}{r_1 + a_1 \cot \delta_1}, \\
2 \delta_2 &= r_1 \cos \theta_1, & \cos \delta_2 &= \frac{\sin \delta_1}{\cos \theta_2}, & r_2 &= \frac{a_w - r_1 \sin \theta_1}{\sin \theta_2}.
\end{align*}
\]

The parameters of the pitch configurations of orthogonal hypoid gears and gears of type Spiroid [1, 2] are calculated by the relations given in (2.3). The pitch surfaces and circles are visualized in Fig. 2. The pitch configurations, shown in Fig. 1 and Fig. 2, are “gearing primitives” and they are applied to the synthesis of Helicon and Spiroid gear drives. These transmissions are designed as alternative mechanical driving of robot’s hand [3].

3. Technical application

The future humanoid robots will execute various complicated task by communicating with human users. Such robots will be equipped with anthropomorphic multi-fingered hands, which are similar to the human hands. The main future purpose of such humanoid robot’s hand is to replace the human presence in dangerous tasks in fields such as: industrial manufacturing, space, sea-bed and so on. One other future application of the anthropomorphic robot hand is its use as a prosthetic for the handicapped people. Hence, the requirements to such hands are to obtain characteristics as accuracy, smoothness, exactness.

The purpose of the realized study is to replace bevel gear with straight teeth: \((m = 0.5\ mm; \ i_{12} = 4; \ z_1 = 10; \ z_2 = 40)\) with kinematically and strength equivalent hyperboloid gear of type Helicon and Spiroid, which is characterized with increased number of simultaneously contacting teeth (see Fig. 3a, b).

![Fig. 3. CAD design of spatial hyperboloid gear drives (pinion teeth \(z_1 = 8\); gear teeth - \(z_2 = 32\); tooth module - \(m = 0.5\ mm\); offset - \(a_w = 3.25\ mm\)) of type: a) Helicon; b) Spiroid](image)
4. Conclusion

The present paper gives a brief survey of the pitch contact point approach used to design mathematical models of spatial transmission synthesis. This approach has been applied in the Institute of Mechanics of Bulgarian Academy of Sciences to model different types of hyperboloid gears with arbitrarily skewed axes. The application of the pitch contact point model of spatial gears synthesis in Spiroid and Helical gears is also illustrated. Algorithms and computer program for design of Spiroid and Helicon gears are elaborated. The above gear pairs are practically realized as gear drives. This confirms the model plausibility and enables specific skew axes gear sets to synthesis.

The realized study is an illustration of the application of the approach to the synthesis of hyperboloid gear pairs upon a pitch contact point. Here is presented an attempt to create an adequate gear drive applicable to the design of the robot's hand. This robot is an innovation product of Mechanical Engineering Department at Gifu University, Japan.

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