ABSTRACT. The present study extends the procedure proposed by the authors in [1] and [2] for optimisation of the geometrical shape of arch dams subjected to static loads by taking into account the system’s dynamic characteristics and response to earthquake excitations. The dam shape geometrical parameters whose values are varied in order to satisfy a set of optimisation criteria are referred to as optimisation factors. A reasonable optimisation can be achieved by considering fifteen optimisation factors. The optimisation criteria include both a condition for a minimal volume of the dam body concrete and a condition for maximum dam safety. By means of a rational utilisation of the proposed procedure and a judicious selection of the factors, a significant reduction of the computing effort required for definition of the optimal shape can be achieved. The stress state caused by strong ground motions of Design Earthquake Level is defined by means of the spectral method and is superimposed to the usual load combination of self-weight and hydrostatic pressure for Maximum Impoundment Water Level. The optimal shape of the dam with
minimum concrete volume is then defined by comparing the shapes for which the safety criteria are met under usual and unusual load combination. The stress and strain fields, as well as the dynamic characteristics of the dam are defined by means of the Finite Element Method. Based on the proposed procedure, a computer program has been developed and used for shape optimisation analysis of existing and planned arch dams.

**KEY WORDS:** arch dam, optimisation, earthquake excitation

### 1. Introduction

The arch dams are the only ones among all dams that are able to transfer compressive forces to the abutments of the valley in which they are built. This ability is due to their shape, the most important aspect of which is its horizontal curvature. Thanks to this, the arch dams present a high level of safety against overturning and a more advantageous utilisation of the concrete as a construction material whose tensile strength is considerably lower than its compressive strength. If the geological and the topographical conditions are favourable to an arch option of the planned dam, it can be expected that this will be the most economical option among all considered.

However, the geometrical shape defined during the initial design phase is not always the best one from technical and economical point of view. The best shape should be defined by means of optimisation studies, which employ a set of structural safety and minimal cost criteria. The procedure proposed by the authors in [1, 2] is further developed here in order to take into account the dam site seismic hazard.

### 2. Taking into account the dam site seismic hazard in the dam shape optimisation procedure.

An optimisation procedure for defining the shape of arch dams, in which the usual load combination of self-weight and hydrostatic pressures is only considered, has been presented by the authors in a series of articles and reports. A numerical solution has been proposed in the assumption that the site, the topographic, and the geological conditions exist objectively and do not change during the construction and the operation of the dam. During the optimisation, the co-ordinates of the points of the centreline of the dam base on the river flanks remain constant and the same as the ones defined by the initial design. If it is found that a better solution could be achieved by a different option for the coordinates of the mid-points, then the coordinates could be changed and the optimisation procedure should be repeated.

The optimisation procedure consists in performing a series of FEM analysis by varying the optimisation factors that define the shape of the dam. An automated selection of solutions for the stress states that meet the safety criteria, but do not yield excessively high safety factors, is performed. Among the so-selected solutions, the optimal solution is assumed to be the one of least concrete volume.

The seismic hazard of the site is considered in the optimisation procedure after the definition of the dam stress state caused by the static loads. The procedure consists of defining the stress state due to seismic impact by means of the spectral method. The so-obtained seismic stress state is then superposed to the static stress
Arch Dam Shape Optimization Procedure

state. Since the seismic stresses obtained by the spectral method possess only magnitude but no sign, in the present procedure the superposition is performed with both negative and positive sign to the respective static stress; then the envelopes of the maximum and minimum principal stresses are calculated.

The selection and the comparison of the solutions satisfying the optimisation constraints are performed as in the optimisation procedure for static loads.

3. Optimisation factors

The shape of an arch dam has two basic characteristics: curvature and thickness. Both the curvature and the thickness change in horizontal and vertical directions. There exists no universal functional law for an arbitrary shape of an arch dam, based on independent parameters that could be defined by a functional. Most of the proposed arch dam optimisation procedures are based on some assumptions for the selection of the shape of the dam [3, 4]. In this way, the variation of the curvature and the thickness are defined by means of parabolic, elliptic, trigonometric, exponential, logarithmic, etc. laws. The number of the independent basic parameters varies quite a lot by the different approaches.

In the present study, it is assumed that an acceptable optimisation for the engineering practice can be achieved by means of fifteen independent basic parameters, which are referred to as optimisation factors. The following main assumptions are made: the axis of the highest vertical section is a circular arc, and the axes of the horizontal sections are elliptical arcs. The thicknesses of the vertical and the horizontal sections vary by a degree-three polynomial of the vertical coordinate. For these assumptions, the optimisation factors that are varied in the optimisation procedure are:

1. \( R \): Radius of the circular arc of the highest vertical section.
2. \( Tr \): Translation of the highest vertical section in the along-valley direction.
3. \( Cr \): Overhanging of the crown.
4-7. \( Dc \): Coefficients of a degree-three polynomial defining the variation of the thickness of the highest vertical section.
8-11. \( El \): Coefficients of a degree-three polynomial defining the variation of the ratio of the horizontal sections ellipse semi-axes.
12-15. \( Da \): Coefficients of a degree-three polynomial defining the variation of the thickness of the horizontal sections.

The optimisation factors influence the overall shape of the arch dam differently. The first seven factors, which are related to the location, the shape and the thickness of the highest vertical section, are the most important ones regarding the dam shape. They define the safety against overturning of the dam blocks for empty reservoir conditions during the construction period. Among them, the variation of the radius of the highest vertical section and the overhanging of the crown cause the most significant change of the overturning factor of safety.

The translation of the highest vertical section downstream or upstream has an additional important impact. This factor allows for increase or decrease of the central angle of the horizontal sections, which leads to change of the stress state of the dam
Grigorov, Tasev, Tzenkov, Fanelli, Gunn

and its foundation. An upstream translation improves the stress state but causes also an increase of the dam body concrete volume and can lead to excessively inclined abutment thrusts. Therefore, such translation should be used only in case a significant change of the stress state proves necessary.

Some interesting observations could be made regarding the thickness of the highest vertical section. The results of the present optimisation studies indicate that increasing the thickness of the dam in its lowest part does not always lead to a meaningful improvement of the stress state in this zone. Therefore serious engineering judgment should be exerted in accepting solutions leading to considerable increase of the thickness in the lower third of the dam.

It is worth noting that the change of the ratio of the horizontal sections ellipse semi-axes cannot lead to a considerable change of the dam body stress state; however, its variations could be used for directing in a favourable way the forces transmitted from the dam to the abutments.

4. Optimisation criteria

The values of the optimisation factors are varied in order to define an optimal shape of the arch dam. Only the solutions that satisfy the safety criteria without excessively high factors of safety are retained. Some authors refer to the safety criteria as constraints [4]. The safety criteria concern the safety against overturning of the dam cantilevers during the construction period, the safety against stresses at the base joint higher than the tensile strength of the joint, and the safety against tensile and compressive stresses in the dam body higher than the respective allowable stresses. The main optimisation criterion is the minimum volume of the concrete in the dam body.

5. Numerical investigations

A numerical example has been carried out on a concrete arch dam of topographical and geometrical parameters similar to those of the Emosson Arch Dam in Switzerland. The dam height is 155.00 m; the thickness of the central vertical section is 38.26 m at the base and 9 m at the crown. The design strength of the mass concrete, which has been used as criteria for the shape optimisation, has been calculated according to [5] and has the following values: compressive strength of 8.3 MPa for usual load combinations and 19.2 MPa for unusual load combination with earthquake and tensile strength of 1.6 MPa for usual load combination and 4.6 MPa for unusual load combination with earthquake.

The optimisation parameters and the dam body concrete volume for the shape of the dam defined in the original design are given in the first row of Table 1; the corresponding values defined by means of the proposed optimisation procedure are given in the second row of the table. As it can be seen, the volume of the dam body defined by the present optimisation is 177417 m$^3$ less than the original volume, i.e. 17.4 % less. (The notation in Table 1 is given in Section 3.)
Table 1. Optimisation factors and dam body concrete volume

<table>
<thead>
<tr>
<th>No</th>
<th>$R$ [m]</th>
<th>$Tr$ [m]</th>
<th>$Cr$ [m]</th>
<th>$Dc_1$ [m]</th>
<th>$Dc_2$ [m]</th>
<th>$Dc_3$ [m]</th>
<th>$Dc_4$ [m]</th>
<th>$El_1$</th>
<th>$El_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>205.0</td>
<td>0.00</td>
<td>-2.80</td>
<td>38.26</td>
<td>23.47</td>
<td>19.13</td>
<td>9.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>230.0</td>
<td>0.00</td>
<td>2.50</td>
<td>32.26</td>
<td>22.22</td>
<td>15.63</td>
<td>5.00</td>
<td>0.20</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Although the concrete volume defined as a result of the present optimisation is significantly less than the volume of the original design, the stress state of the dam for usual and unusual load combinations does not change much (Fig. 1 and Fig. 2).

Fig. 1. Principal stress $\sigma_1$ in the central vertical section
a) Original design shape, usual load combination; b) Optimised shape, usual load combination; c) Original design shape, unusual load combination; d) Optimised shape, unusual load combination.

6. Conclusion

The following conclusions can be drawn based on the present numerical investigations:
- The most important parameters defining the safety against overturning during the construction period are the radius of the circular arc of the highest vertical section and the overhanging of the crown.
The stress state of the dam depends mostly on the thickness and its variation in the horizontal and the vertical directions. It is recommended not to change significantly the thickness in the vertical direction within the lower third of the dam.

- Translation of the highest vertical section in the along-valley direction can be employed if it is necessary to improve the maximal principal stresses.
- The ratio of the horizontal sections ellipse the semi-axes influences insignificantly the dam body stress state; however, it can have a favourable effect on the direction of the forces transmitted from the dam to the abutments.
- The proposed optimisation method can be further developed and improved mainly with regard to the computer processing of the solutions for the dam shape and the search of the optimal one.

**REFERENCES**


