MODELING OF LANDING GEAR STRUT BY COMPONENT MODE SYNTHESIS

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ABSTRACT. In this paper nonlinear finite element of landing gear strut is presented. Element is derived by modeling bending and torsional stiffness and mass properties of main parts as beams using component mode synthesis and final matrices are evaluated by static condensation. Damping and elastic properties in longitudinal direction are evaluated by using of nonlinear equations of oleo-gas shock-absorbers and experimental data.

KEY WORDS: finite elements, component mode synthesis, nonlinear dynamics

1. Introduction
Multibody dynamics simulation of flexible bodies plays a key role in design and production of real structures and machines nowadays. But it requires a lot of time and computing resources for complex structures with thousands degree of freedom (d.o.f.) and efforts are directed towards reducing them. Usually complex structures are replaced by models with few d.o.f. that preserve main dynamical properties of initial models.

The goal of this study is to show how to substitute complex model like landing gear strut with simple two node finite element for multibody dynamics simulation by using component mode synthesis. Obtained finite element model is applicable for using with co-rotational method for nonlinear analysis.

Landing gear strut of helicopter Mi-8 is used as example.

2. Landing gear strut model
Landing gear strut of Mi-8 [1] consists of two stages: low pressure stage (A on Fig. 1) and high pressure stage (B on Fig.1). Every stage has piston (I and IV) and cylinder (II and V). Cylinder II of stage A and piston IV of stage B are tightly connected by element III. Cylinders are filled with fluid in lower chambers and with air in upper chambers. When piston is moving down in respective cylinder fluid is...
moving from lower to upper chamber and air in upper chamber is compressed. Moving fluid is passing through orifices and energy is converted to heat.

Elements of strut can be modeled by beam finite elements (Fig. 2). Since piston and cylinder have two contact points they must be modeled by three nodes beam elements. When piston is moving in cylinder contact point (mid node) is also moving. To describe accurately properties of cylinder-piston component three mutual positions are regarded (Fig. 3): fully extended position, middle position and fully compressed position. For every position stiffness and mass matrices are assembled and matrices for all other intermediate positions are calculated by quadratic interpolation element by element.

After generating matrices for both cylinder-piston components they are assembled together with matrices of connecting element III and unneeded internal nodes are eliminated (Fig. 2) by static condensation [3]. It must be noted that axial
stiffness is not regarded because flexibility in this direction is dominated by compressed air in chambers and is described later.

3. Synthesizing beam elements

Process is starting by creating CAD models for every element. Such model of piston I is shown on Fig. 4. In CAD model there are defined zones that correspond to two end nodes and three zones that correspond to three positions of middle node. Next step is generating finite elements that determine initial finite element (FE) model for piston I. Such initial FE models are created for every element of strut. They consist of hundreds or thousands finite elements and nodes.

Variant of component mode synthesis (CMS) method is used to convert initial FE models into two or three nodes beam elements. This variant of CMS is described in details in [6, 7] and here it will be described briefly.

Nodes in the initial FE model are divided into internal nodes and dependent nodes. Dependent nodes (also known as slave nodes) are nodes from defined zones in CAD model and remaining nodes are internal nodes. Master nodes are nodes of synthesized beam element and they may not exist in the initial FE model. Dependent nodes from one zone form one group and their displacements depend on displacements of corresponding master node (Fig. 5) according to classical beam
theory. In Fig. 6 are shown displacements of slave nodes due to two translations and one rotation of corresponding master node in plane.

Fig. 5. Master-slave method. Fig. 6. Displacements of slave nodes in plane.

Displacements of nodes in initial FE model $\mathbf{u}$ and displacements of beam nodes $\mathbf{\bar{u}}$ are related by

$$\mathbf{u} = \mathbf{W}\mathbf{\bar{u}},$$

where $\mathbf{W}$ is transformation matrix. Columns of $\mathbf{W}$ have simple physical meaning – they are displacements of nodes in initial FE model due to unit displacements in master nodes. Using FE expression for energy transformation of matrices from initial FE to beam model can be obtained

$$\mathbf{K} = \mathbf{W}^T\mathbf{K}\mathbf{W}, \quad \mathbf{M} = \mathbf{W}^T\mathbf{M}\mathbf{W},$$

where $\mathbf{K}$ and $\mathbf{M}$ are stiffness and mass matrices in initial FE model, $\mathbf{K}$ and $\mathbf{M}$ are stiffness and mass matrices in beam model.

Dynamical properties of synthesized beam elements can be improved by adding to transformation matrix $\mathbf{W}$ vibration modes from initial FE model [4-8].

4. Axial properties of piston-cylinder components

Axial flexibility of piston-cylinder components are dominated by compressibility of air in chambers. Axial force $P$ due to compressed air depends on initial pressure $p_0$ in chamber and varies nonlinearly with axial displacement $u$ [2]:

$$P(u) = A_A p_0 \left( \frac{V_0}{V_0 + A_A u} \right)^n,$$

where $V_0$ is initial volume ($u = 0$) of air, $A_A$ is area of piston section and $n$ is polytropic index. Tangential stiffness is [3]

$$k = \frac{dP(u)}{du},$$

and tangential stiffness matrix for piston-cylinder component in axial direction is

$$\mathbf{K}_t = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}.$$
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Relation (3) is theoretical and can be assessed fully by experimental tests and numerical evaluation of first derivatives in (4). Different way is by differentiating (3)

\[
k = \frac{dP}{du} = \frac{A^2 p_0 \left( \frac{V_0}{V_0 + A_i u} \right)^n}{V_0 + A_i u}
\]

and measuring individual parameters. This method is less accurate because of difficulties in measuring polytropic index \( n \).

Damping properties in axial directions depend on fluid properties, orifice geometry, piston position and speed, motion direction. Theoretical force is [2]

\[
Q(u, \dot{u}) = \frac{\rho_f}{2} \frac{A_i^3}{\mu A_i(u)} \dot{u}^2 \text{sgn} \dot{u},
\]

where \( \rho_f \) is fluid mass density, \( A_i \) - area of orifice, \( \mu \) - coefficient that depends on fluid viscosity and orifice geometry.

Tangential damping matrix is [3]

\[
C_t = \begin{bmatrix} c & -c \\
-c & c
\end{bmatrix},
\]

where

\[
c = \frac{dQ(u, \dot{u})}{du}.
\]

Relation for tangential force in (7) is more difficult to assess because it depend on two state variables: position \( u \) and speed \( \dot{u} \) of piston. Thus it needs more experimental tests. The other way by differentiating (7)

\[
c = \frac{dQ}{du} = \rho \frac{A_i^3}{\mu^2 A_i(u)} \dot{u} \text{sgn} \dot{u}
\]

and measuring individual parameters is far less accurate because there are many orifices, direction valves, metering pin that change orifice area and etc. It is applicable only in simple cases with only one orifice.

5. Conclusions

Method for modeling complex structures and converting this models into simple finite element models with few d.o.f. is presented. It is based on component mode synthesis and experimental tests and evaluation of some properties. Landing gear strut of helicopter Mi-8 is used as example.

REFERENCES