MESO-MACRO APPROACH FOR INVESTIGATION OF THE RUPTURE OF FLEXIBLE TEXTILE COMPOSITES

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ABSTRACT. In the resent work some experimental results concerning the development of the rupture in specimens of flexible textile composites under uniaxial tension are presented. The boundary conditions when the rupture does not propagate are defined. Criteria characterizing the resistance of the composites investigated against propagation of local defects are suggested.

KEY WORDS: flexible textile composites, woven composite material, coated fabric

1. Introduction
The existing theories for calculation of membrane structures from flexible textile composites are based mainly on the assumption that the destruction begins in the areas of maximum tension. Our investigations on real structures show that the membrane structures compromise themselves in areas which do not coincide with them. The behavior of textile composites when rupture appears is no enough investigated. That is a quite complicated process, which depends on a lot of factors – structure of the fabric, stiffness of the material, properties of the cover, etc., which
appear in different degree in dependence of the stress state. The means of the
destruction mechanics cannot be directly applied for description of the process of
rupture in this type of materials.

In the resent work some experimental results concerning the development of
the rupture in specimens of flexible textile composites under uniaxial tension are
presented. The boundary conditions when the rupture does not propagate are defined.
Criteria characterizing the resistance of the composites investigated again
propagation of local defects are suggested.

2. Experimental work

The investigated composite is polyester textile (satin 9×9), coated by
plastic PVC. This kind of material is widely used in construction of many types
membrane structures. All specimens have width 4cm. The obtained stress-strain
diagrams are presented in Fig. 1.

![Stress-strain diagram](image)

Fig. 1

Observations on the alternation of the meso-structure of the investigated
material by uniaxial tension and increasing load are carried out. Microphotography
pictures of the cross section of the composite in the direction of the weft are
presented in Fig. 2 (magnification 6:1).

![Microphotography pictures](image)

Fig. 2

b) $T = 9.5 \text{ N/cm}$  c) $T = 60 \text{ N/cm}$  e) $T = 235 \text{ N/cm}$

Investigations for determination of the area and the way of influence of
preliminary made cut are carried out. The specimens are with width 4cm and the
distance between the maxillae is approximately 12cm. The cut has width $a_0 = 6.5\text{mm}$ (Fig. 3). An in-plane sensitive digital speckle-pattern interferometer by very slow rate of loading (see [3, 4]) is used for determination of the deformations in the specimen (Fig. 4).

3. Analysis of the experimental results

The analysis of the experimental data shows that in the initial stage of low rate of loading only the area outside of the cut works, while in the area of the interrupted fibers only displacement is observed (Fig. 3b and c). It could be accepted that in this stage the interaction (the friction) between the fibers of the wrap and the weft is neglecting small. So the area of the influence of the cut is spread out to its width, i.e. $a = a_0$. In the second stage, the forces of friction between the fibers increase according to the increasing of the loading, so there is a transferring of the forces from the loaded to the unloaded areas. The damaged area becomes with width $a > a_0$ (Fig. 3d and e) and in the direction of the tension is restricted from the planes of lowest resistance of the composite (Fig. 5a). Here $\alpha_0 \approx 45^\circ$ represents the angle of lowest resistance, i.e. the direction of lowest Young’s modulus (see [2]). By increasing of the load more than a certain rate the sizes of the damaged area remain constant but the forces in the fibers increase until the rupture of the first fiber (Fig.
3f). A typical chart of the relation between the load and the size \( a \) is presented on Fig. 5b (see [2]).

![Diagram](image)

**Fig. 5**

### 4. Mathematical model

From the presented analysis it could be concluded that for determination of the influence of the cut two damaged areas is necessary to be considered – one over and under the cut and the second in the outer aside of it (Fig. 6a). Having in mind the meso structure of the material there are no areas in which only the fibers in one direction be affected. So the number of the affected fibers is equal in both directions. It suffices only one interrupted fiber to be considered.

The relations obtained in [1] are used for determination of the forces in the fibers. The force in a fiber in the direction of the load than is

\[
N_\infty = \frac{Tk \cos \phi}{m(1 + k)},
\]

where \( T \) is the rate of load per unit width, \( \phi \) is the angle between the fiber and the middle plain of the composite, \( m \) is the number of fibers per unit width and \( k = (s_1E_1)/(s_2E_2) \). Here \( E_1 \) and \( E_2 \) are the elastic modulus of the fiber and the matrix respectively, \( s_1 \) and \( s_2 \) are the percentage of fiber and matrix for the cross section of the material.

Then the friction force between the fibers in both directions \( N_{f1} \) is

\[
N_{f1} = N_{fibre}(1 - e^{-2\mu_1\phi}),
\]

where \( N_{fibre} \) is the force in the corresponding node \( \mu_1 \) is the friction coefficient between the fibers.

So for the force in the fiber after the first node in the damaged area over (under) the cut is obtained

\[
N_{1, fibre} = N_\infty - N_{f1} = N_\infty - N_\infty(1 - e^{-2\mu_1\phi}) = N_\infty e^{-2\mu_1\phi}.
\]

After the second node the force is respectively

\[
N_{2, fibre} = N_{1, fibre} e^{-2\mu_2\phi} = N_\infty e^{-2\mu_1\phi} e^{-2\mu_2\phi} = N_\infty e^{-4\mu_1\phi}.
\]
So after the $i$-th node one obtains:

$$N_{i,\text{fibre}} = N_{i} e^{-2\mu_{i}\varphi}$$

For determination of the force at the boundary of the cut we can use the condition that the friction between the fiber and the matrix must be lower than the admissible, or we can use the inequality

$$N_{i,\text{fibre}} e^{-2\mu_{i}\varphi} \leq \frac{\mu_{2}T_{\text{adm}}}{m},$$

where $T_{\text{adm}}$ is the admissible force per unit, $\mu_{2}$ is the friction coefficient between the fiber and the matrix. Then for one fiber it can be written

$$N_{i} e^{-2\mu_{i}\varphi} \leq \frac{\mu_{2}T_{\text{adm}}}{m}.$$

So for the number of the nodes in the affected area one obtains

$$n \geq \frac{m_{\text{rup}}}{2m} \cdot \frac{1}{2\mu_{i}\varphi} \ln \frac{N_{i}m}{T_{\text{adm}}\mu_{2}}$$

where $m_{\text{rup}}$ is the number of the ruptured fibers.

It can be illustrated by an in-plane sensitive digital speckle-pattern interferometer (Fig. 4) that the displacements in the damaged area are distributed in curves, which sufficient accurately can be approximated by quadratic function. On the other hand the sum of the forced distributed in the area over (under) the cut $\frac{N_{i}m_{\text{rup}}}{2}$ must be equal to the sum the additional forces in outside area $\sum \Delta N_{i}$ (Fig. 6c). For the force in the first fiber (nearest to the cut) $N_{1}$ one obtains:

$$N_{1} = N_{o} \left(1 + \frac{3a_{0}m_{\text{rup}}n}{2m}\right).$$

As a criteria for stability of the cut then we have that $N_{1} \leq N_{\text{adm}}$, where $N_{\text{adm}}$ is the admissible force in the fiber.
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5. Numerical example

The suggested approach is applied for the investigated composite – polyester textile (satin 9 × 9), coated by plastic PVC. In the direction of the weft \( T_{adm} = 650 \text{ N/cm} \) (i.e. \( N_{adm} = 76 \text{ N} \)). The friction coefficients are \( \mu_1 = 0.35 \) and \( \mu_2 = 0.20 \). The number of the ruptured fibers is \( m_{rup} = 6 \). The results are given in Table 1.

<table>
<thead>
<tr>
<th>( T ) [N/cm]</th>
<th>( \phi ) [rad]</th>
<th>( N_{\infty} ) [N]</th>
<th>( n ) (obtained)</th>
<th>( n ) (accepted)</th>
<th>( N_1 ) [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>0.1248</td>
<td>14.33</td>
<td>-0.0304</td>
<td>0</td>
<td>14.33</td>
</tr>
<tr>
<td>195</td>
<td>0.0634</td>
<td>21.62</td>
<td>3.0279</td>
<td>3</td>
<td>54.17</td>
</tr>
<tr>
<td>210</td>
<td>0.0596</td>
<td>23.29</td>
<td>3.8271</td>
<td>4</td>
<td>81.27</td>
</tr>
<tr>
<td>240</td>
<td>0.0522</td>
<td>26.63</td>
<td>5.5814</td>
<td>6</td>
<td>133.22</td>
</tr>
</tbody>
</table>

From the pictures in Fig. 4 it is obvious that \( T = 210 \text{ N/cm} \) is the upper limit, after which the cut propagates and becomes with larger sizes. So, the comparison with the results of the physical experiments (see Fig. 4f and the third row of the table) shows sufficient exactness of the suggested mathematical model.

6. Conclusion

The created simplified model for determination of the areas of damage in flexible textile composites gives adequate results and can be applied in practice. The approach gives an approximate evaluation of the crucial rate of the load in appearance of macro damage in the structure.

REFERENCES