APPLICATION OF EXACT GOVERNING DIFFERENTIAL EQUATION OF MOTION OF 6NVD-18 SHIP ENGINE FOR DETERMINATION OF ITS DYNAMIC BEHAVIOUR

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ABSTRACT. In the previous direct method application to engines the general governing differential equation of motion for the mechanical part of an arbitrary combustion engine dependent on the crankshaft angle was obtained. In the present paper the governing differential equation of motion for the crankshaft of 6NVD-18 four-stroke ship engine that is used in marine and river vessels as a main engine or as an auxiliary diesel-generator is derived. The obtained governing equation of motion contains full kinematical and dynamical information on the closed kinematical loops of crankgears assembling the mechanical part of the engine. The general type of governing equation for a crankshaft of a combustion engine is corroborated by this example. Numerical solution of the equation utilizing the indicator pressure reference diagram of the engine was used for determination the crankshaft rotation irregularity. The obtained results of calculation were compared to the ordinary data obtained from the numerous measurements and tests carried out and studied during the courses in Marine Engineering majors at TU-Varna.

KEY WORDS: combustion engine, dynamics, governing equation

1. Introduction
The improvement of contemporary combustion engine operation is related to precise determination of the crankshaft rotation degree of irregularity, the external unbalance with respect to the inertia net-force and net-moment of rotating and reciprocating masses, reaction forces at bearings, vibration of engine links, etc.
In the theory of machines, however, the traditional way of consideration such complex problems is not based on the analysis of closed dynamical equations of motion of an entire mechanical system, as is common for the ordinary dynamical problems in mechanics. Some simplified analytical models that are away from the complexity of the real object instantaneous motion are rather available instead. Apparently this is due to the closed kinematical loops of slider-crank mechanisms assembling a combustion engine that highly complicates derivation of the governing differential equation of an engine crankshaft rotation.

For systematic solution of the above mentioned problems a new approach based on the method [1] for derivation of governing dynamic equations for multibody systems is offered. In the previous applications of the method to the operation of combustion engines [2, 3, 4] the governing differential equations of motion for the mechanical part of various combustion engines dependent on the crankshaft angle were obtained. All of them prove the universal type of the received governing dynamic equations of a crankshaft rotation. Some general features of the proposed method are as follows.

2. Basics of the direct method for multibody systems

It is well known [5] that for a system of rigid bodies subjected to holonomic ideal constraints the following notation of D’Alembert- Lagrange’s principle is valid:

\[
\sum_{k=1}^{N} \left( \vec{F}_k - M_k \ddot{r}_{ck} \right) \cdot \delta \vec{r}_{ck} + \left( \vec{M}_{ck} - \frac{d\vec{K}_{ck}}{dt} \right) \cdot \delta \vec{\pi}_{ck} = 0,
\]

where \( N \) is the number of rigid bodies constituting the mechanical system. The mass \( M_k \) of a rigid body number \( k \) and its angular momentum \( \vec{K}_{ck} \) about the body mass center \( C_k \), \( k = 1, \ldots, N \) are introduced in the parentheses above. By means of \( \vec{F}_k \) and \( \vec{M}_{ck}, k = 1, \ldots, N \) all active forces and moments exerted to the body \( k \) are denoted respectively, as external and as internal. Vector \( \delta \vec{r}_{ck} \) of the position vector \( \vec{r}_{ck} \) of point \( C_k \) variation and \( \delta \vec{\pi}_{ck} \) vector of the body \( k \) angular orientation variation with respect to an inertial reference frame are presented in principle (2.1).

In the principle by substitution the position and orientation vectors dependent on the generalized coordinates \( q_j, j = 1, \ldots, s \) of the system and by cancellation the independent generalized coordinate variations following common algorithms of analytical mechanics for these operations, we can receive the equations of motion as:

\[
\sum_{k=1}^{N} \left[ Q_j(F_k) + Q_j(\phi_k) + Q_j(M_{ck}) + Q_j(M_{ck}\phi_k) \right] = 0, \quad j = 1, \ldots, s
\]
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where:

\[ Q_j^{(F_k)} = \ddot{F}_k \frac{\partial \vec{r}_{ck}}{\partial q_j} ; \]

\[ Q_j^{(\phi_k)} = -M_k \ddot{\vec{r}}_{ck} \frac{\partial \vec{r}_{ck}}{\partial q_j} ; \]

\[ Q_j^{(M_{ck})} = M_{ck} \ddot{\vec{r}}_{ck} \frac{\partial (\delta \vec{r}_k)}{\partial q_j} ; \]

\[ Q_j^{(\phi_{ck})} = -\frac{d\dot{K}_{ck}}{dt} \frac{\partial (\delta \vec{r}_k)}{\partial q_j} \]

are the generalized forces of the applied and inertia forces, the applied and inertia moments respectively acting on the bodies of a mechanical system.

It can be observed from the above, that the suggested method for the dynamic governing equations derivation come down to series of easily executed operations of multiplication and summation for the generalized forces formation with their consecutive general addition. By these means, no any extra operations are necessary like the obtaining of kinetic energy or its derivatives. The governing equations of motion of a system can be received directly by the suggested method.

Realising the method on computer, these operations can be effectively executed by means of appropriate available packages for symbolic operations like MATHEMATICA, MAPLE, REDUCE, MATLAB, MUPAD, MAXIMA, etc.

For example, the necessary input data used for the direct method application are as follows below. Let us note that on the basis of the proposed direct method the governing differential equations of motion for systems of flexible bodies can be successfully derived as well, as it is shown in [1].

3. Input data for a crankgear

On the base of direct method the computer program EQUATION is composed in the environment of MAPLE package. Let us present the input of the program for a crankgear, which appearance is depicted in the following figure.

![Fig. 1. Kinematical scheme of the crankgear](image)

**Crankshaft data**

Inertia moment \( J_z \) of the body: \( I_{C_1} \);

Dependency for the angle of turn: \( \varphi \);

Dependency for Mz torque: \(-M\);

**Piston data**

Mass of the body: \( m_3 \);

Dependency for \( Xc \): \( l_1 \cos \varphi + l_2 \cos \psi \);

Dependency for force Fx: \(-F_1\);

**Connecting rod data**

Inertia moment \( J_z \) of the body: \( I_{C_2} \);
 Dependency for Xc: \( l_1 \cos \phi + b \cos \psi \);  
Dependency for Yc: \( (l_2 - b) \sin \psi \);  
Dependency for angle KSI: \( \psi = \arcsin \left( \frac{l_1}{l_2} \sin \phi \right) \);

If there are more pistons and connecting rods in a combustion engine, the similar data are inputted for them as well. For instance, 6NVD-18 four-stroke ship engine comprises six crankgears. Let us apply EQUATION program to receive 6NVD-18 crankshaft governing equation.

4. General description of 6NVD-18 ship engine

Diesel combustion engine 6NVD-18 is a four-stroke ship engine that is used in marine and river vessels as a main engine or as an auxiliary diesel-generator. Diameter of pistons is of 125 mm, while the piston stroke is of 180 mm. Mass of a piston is of 1.5 kg. The crankshaft radius is of 47.5 mm and a connecting rod length is of 157.9 mm. It has 5 kg mass. In front of the crankshaft a fly-wheel is attached. Its moment of inertia together with the crankshaft in respect to the rotation axis is of 100 kgm². The consecutive order of the pistons’ ignition is 1-2-3-4-5-6, while the adjacent pistons are dephasing by 120°. Approximate relation for the indicator pressure reference diagram that well complies with test data [6] is presented in Fig.2.

![Fig. 2. Approximate relation for an indicator pressure reference diagram](image.png)

5. Calculation results

After substitution for all numeric and symbolic data of 6NVD-18 ship engine into EQUATION program the following governing differential equation of motion for the crankshaft angle \( \phi \) of rotation is received. Note that all calculations are in SI.
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\[
[100.0746 + 0.5343 \cdot 10^{-2} \cos(6\varphi)] \cdot \dot{\varphi} + 0.5025 \cdot 10^{-4} \sin(6\varphi) \cdot \varphi^2 = \\
0.475 \cdot 10^{-1} [\sin(\varphi)F_1 - \sin(\varphi + \frac{\pi}{3})F_2 - \sin(\varphi - \frac{\pi}{3})F_3 + \\
\sin(\varphi)F_4 - \sin(\varphi + \frac{\pi}{3})F_5 - \sin(\varphi - \frac{\pi}{3})F_6] - \\
-[1 + 0.2133 \cdot 10^{-3} \cos(6\varphi)]M
\]

(5.1)

where \( F_i, i = 1, \ldots, 6 \) are net gas forces over the respective piston, \( M \) is a net resistant moment applied to the crankshaft.

The first term at the right hand side of (5.1) containing forces in brackets represents an active moment \( M_{\text{active}} \) applied to the crankshaft. Time-chart of the active moment for 5 crankshaft turns is presented in Fig. 3 below.

![Time-chart of the active moment for 5 crankshaft turns](image)

The highly variable active moment results in the crankshaft rotational irregularity that means continuous deviation from the constant angular velocity of rotation during engine’s work. According to the Ship Register Regulations [6] the irregularity \( \delta \) of the crankshaft angular velocity of rotation for screw driving ship engines must be roughly of \( \delta=1/150 \) (or \( \delta=0.67\% \)), while for the ship engines working as diesel-generators it is of \( \delta=1/250 \) (or \( \delta=0.4\% \)).

After solving the obtained (5.1) ODE with an appropriate numerical method the angular velocity chart depicted in Fig. 4 was received. It appears that the irregularity for 6NVD-18 ship engine is of \( \delta=0.328\% \), that means the fly-wheel of 6NVD-18 engine is slightly oversized and its moment of inertia can be diminished. As a set of new calculations shows the 6NVD-18 engine fly-wheel moment of inertia may be reduced from 100 kgm² up to 85 kgm² without violation of Regulation limits.
6. Discussion

For a combustion engine the direct method gives an opportunity to derive its governing equation of motion that is a tool for engine’s design and development. By the help of this equation the following problems can be solved:

- Change of the crankshaft angle and angular velocity as functions of gas forces in cylinders, an applied resistant moment, geometrical and mass characteristics of the engine parts.
- Determination of the engine’s degree of irregularity and design of the fly-wheel for moment of inertia
- The obtained functions for crankshaft angle rate of change, the acceleration as well, are the necessary foundation for the consecutive determination of the inertia and reaction forces acting during the engine’s work.

REFERENCES


