ABSTRACT. The paper refers to a step by step FEM procedure. The aim is to calculate the critical force of straight bars when plastic deformations arise. An intersectional module of elasticity is used. The constitutive law of the material in the zone of plastic deformation is presented as a two-degree polynom depending on the zone’s length.

KEY WORDS: Finite element method, critical force, plastic deformation, step by step procedure.

1. Introduction.

In [1], an analytical procedure is shown on how to calculate a critical force of a two-body-cut trusses – fig. 1a, having different ratios between the lengths of the strengthened and the weakened zone of the truss, since the type and the proportions of the cross sections varies, while 3 out of the 4 pertaining physical features are being set in the diagram $\sigma(\varepsilon)$ of the material.

This particular diagram represents the growth of plastic deformations which are shown in fig.1b where:

$\sigma_p$ - stress in limit of proportional
$\sigma_R$ - failure stress
$\varepsilon_p$ - strain in limit of proportional
$\varepsilon_R$ - ultimate strain of failure
The analytical equation on is: \( \sigma = \sigma(\varepsilon) \)

\[
\sigma = \sigma_p + E(\varepsilon - \varepsilon_p) - \frac{E}{2\delta}(\varepsilon - \varepsilon_p)^2
\]

where \( \varepsilon_p \leq \varepsilon \leq \varepsilon_R \), and:

\[
\delta = \varepsilon_R - \varepsilon_p
\]

The change of the tangent module within the zone of plastic deformation is:

\[
E(\varepsilon) = \frac{\partial \sigma}{\partial \varepsilon} = E - \frac{E}{\delta}(\varepsilon - \varepsilon_p)
\]

It can be seen that in case of \( \varepsilon = \varepsilon_p \), \( E(\varepsilon) = E \), which makes for a smooth transition from the elastic zone towards the zone of plastic-deformations. If \( \varepsilon = \varepsilon_R \) and \( \delta = \varepsilon_R - \varepsilon_p \) from (1.1) we obtain:
A procedure on studying a critical force on FEM

(1.4) \[ \sigma_R = \sigma_p + E\delta - \frac{E\delta}{2} = \sigma_p + \frac{E\delta}{2} \]

That value does not overlap with \( \sigma_{max} \) of the curve on fig 1b. The contradiction is due to the fact that the unsettled plastic behaviour of the material is determined by 4 independent parameters - \( \sigma_p, \sigma_R, E, \delta = \varepsilon_R - \varepsilon_p \), however the square polynomial (1.1) is being defined only of 3 independent constants.

In [1], with certain approaching, the relationship (1.1) is being formulated as we use:

(1.5) \[ \delta = \frac{2(\sigma_R - \sigma_p)}{E} \]

This case has been explored within the limit of linear-elastic behaviour of material and it has also been explored by FEM [1], [2]. In a subsequent condensation of a net of discrete nods upon the truss of fig. 1a, the equation of buckling, has got to be used:

(1.6) \[ \det [K] - F[K_G'] = 0 \]

where:

\[ [K] = \sum [k^e] \] is global flexible stiffness matrix,
\[ [K_G'] = \sum [k_G^e] \] is global geometric stiffness matrix.

\[ [k^e] = EJ \]

\[ [k_G^e] = \frac{F}{30l} \]

\[ \begin{bmatrix}
12 & 6 & 12 & 6 \\
\frac{l^3}{6} & \frac{l^2}{2} & \frac{l^3}{6} & \frac{l^2}{2} \\
\frac{6}{l^2} & \frac{4}{l} & \frac{6}{l^2} & \frac{2}{l} \\
12 & \frac{6}{l^3} & \frac{12}{l^3} & \frac{6}{l^3} \\
\frac{6}{l^2} & \frac{2}{l} & \frac{6}{l^2} & \frac{4}{l^2} \\
\frac{1}{l^2} & \frac{1}{l} & \frac{1}{l^2} & \frac{1}{l}
\end{bmatrix} \]

\[ \begin{bmatrix}
36 & 3l & -36 & 3l \\
3l & 4l^2 & -3l & -l^2 \\
-36 & -3l & 36 & -3l \\
3l & -l^2 & -3l & 4l^2
\end{bmatrix} \]
As we enter into the zone of plastic deformations, the calculation of the critical parameter \( F \) of (1.6), must be organized as a step-by-step procedure. The changing tangent module \( E(\varepsilon) \) is to be embodied within the stiffness matrix elements \( k^* \), thus in \( [K] \), whereas \( \left[ K_{cr} \right] \) remains invariable.

Another option for critical parameter study within an iteration procedure as the intersectional module of elasticity has been introduced by B. Bankov in [1], yet with no gain of a result expressed numerical. This is what will be done in this paper.

2. Algorithm for calculating the critical parameter of a truss loaded by its axis on FEM while plastic deformation arises and as the intersectional module of elasticity is used.

We shall limit the range of this case, as we shall organize a procedure for calculating of \( F_{cr} \) on FEM of a truss whit a constant square cross-section, i.e. at \( l_1 = l \) and \( l_2 = 0 \) (fig.2) along with discretization of 20 truss finite elements.

Let’s assume as a computation diagram of material (1) to be the one shown as a graph on fig.3. The global matrix of cure is: \( [K(E_1)] = [K_1] \), \( E_1 = tg\alpha_1 \).

\[
\frac{1}{2} \alpha_1
\]

Fig. 2.

From the buckling equation \( \det \left[ K_1 - F(K_{cr}) \right] = 0 \), we gain initial parameter approaching, \( F_{cr} = F_1 \), \( \sigma_1 = \frac{F_1}{A} \), \( \varepsilon_1 = \frac{\sigma_1}{E_1} \). Having the calculated deformation and the logged formula of analytic ratio we do the following calculation \( \sigma'_1 = \sigma(\varepsilon_1) \), \( E_2 = \frac{\sigma_1}{\varepsilon_1} \).

The global stiffness matrix is \( [K_2] = [K_2(E_2)] \) then again we apply the buckling equation: \( \det \left[ K_2 - F(K_{cr}) \right] = 0 \).
A procedure on studying a critical force on FEM

Fig. 3.

We calculate $F_{cr} = F_2$, $\sigma_2 = \frac{F_2}{A}$, $\varepsilon_2 = \frac{\sigma_2}{E_2}$, $\sigma_3 = \sigma(\varepsilon_2)$, $E_3 = \frac{\sigma_3}{\varepsilon_2}$.

$[K_3] = [K_3(E_3)]$ and i.e. whenever eventually obtain: $F_{cr}^{i-1} \approx F_{cr}^i$.

3. Numeric example.

Let for truss of fig.2 we have $l = 2m$, $a = 0.02m$, $A = 0.04m^2$,

$I = 1.333 \times 10^{-4} m^4$, $E = 7.5 \times 10^7 kN/m^2$, $\sigma_p = 20.10^4 kN/m^2$,

$\sigma_R = 34.10^4 kN/m^2$, $\varepsilon_p = 2.67.10^{-3}$, $\varepsilon_R = 34.10^{-3}$,

$\delta = \frac{2(\sigma_R - \sigma_p)}{E} = 3.75.10^{-3}$.

Thus of (1.1) for $\sigma$ we obtain:

$$\sigma = 20.10^4 + 7.5 \times 10^7 (\varepsilon - 2.67.10^{-3}) \left(1 - \frac{\varepsilon - 2.67.10^{-3}}{2.373.10^{-3}} \right)$$

The value of buckling force by Euler’s formula, in the case of elastic behaviour of materials is:

$$F_{cr} = \frac{\pi^2 EI}{l^2} = \frac{3.14 \times 1.333 \times 10^{-4} \times 7.5 \times 10^7}{2^2} = 24649 kN.$$
The results of solutions obtain by step by step procedure described in this paper are shown in Table 1.

<table>
<thead>
<tr>
<th>Iteration №</th>
<th>( F_i [kN] )</th>
<th>( \sigma_i [kN / m^2] )</th>
<th>( \varepsilon_i )</th>
<th>( \sigma_i' [kN / m^2] )</th>
<th>( E_i [kN / m^2] )</th>
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<td>24673.97</td>
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<td>7.5000.10⁷</td>
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<td>-</td>
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</tr>
</tbody>
</table>

4. Conclusion.

Table 1 shows clearly a monotonous approximation to the critical value of the load parameter \( F_{cr} \). We can assume that in or during the 6th iteration, the process has practically ended. Thus, the iteration process, based on the use of the intersectional module \( E_i \), might be applied successfully in the event of zone of plastic deformations with evident inequalities \( \sigma_p < \sigma_R \) (in this case \( \sigma_R = 1.7\sigma_p \)). Further calculations have to find out if \( \sigma_R = k\sigma_p \), at what value of \( k \) the iteration process will either slow down or discontinue, meaning there will be no end-computation.

REFERENCES