MODELING OF SERIAL-PARALLEL STRUCTURES WITH ELASTIC JOINTS FOR LOCAL MICRO AND NANO MANIPULATORS*

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ABSTRACT. In this paper piezo actuated micromanipulators are considered with serial-parallel structure including elastic joints. Such structure allows a preliminary tension of the mechanical system in order to eliminate backlashes and to improve the performance of the piezo-actuators. A kinematics model of a serial-parallel structure for local micro manipulators is build here. A pseudo rigid body approach is used, where elastic joints are modelled as revolute joints. A stiffness model is created to estimate the general stiffness of the manipulator by means of reduction the stiffness of all elastic joints. Two approaches are presented here for preliminary tension of parallel manipulator structure.

KEY WORDS: serial-parallel micromanipulator, elastic joints, stiffness model, tension.

1. Introduction.
There are known micromanipulators with piezo actuators and parallel structure [1] [2]; [3]. Mechanisms with closed kinematic chains are suitable for high-precision tasks in 3D space. The high accuracy of such mechanical systems is due to very high structural stiffness.

To predict the displacements of compliant mechanisms with elastic joints the pseudo-rigid-body-model is commonly used [4]. As a rule it models an elastic joint

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as a revolute joint with a torsion spring attached. The pseudo-rigid-body method is effective and it simplifies the model of compliant mechanisms. To estimate the mechanism stiffness with elastic joints an analytical model is created out taking into account compliances of elastic joints in all axes. The analytical model is describing the relationship between input and output displacements of the mechanism, [5].

The objective of this paper is to create a stiffness model and to develop approaches for tension of serial-parallel structures with elastic joints for micro and nano manipulators with application in cellular technology, microelectronics, chemistry etc.

2. Kinematic model of serial-parallel structures for local micro and nano manipulators.

Investigated structures are serial-parallel structures including basic link 0 and some other links 1, …, n connected in between in a serial chain. The end-effector M is situated in the end link n of this chain, which moves in a v operation space. The driving chains A₁, …, Aₘ, with number m, are attached to the basic link 0 and to the end link n forming parallel chains [6] as it is shown in Fig.1. The type of the kinematics joints is not shown in Fig.1, as they can be elastically or kinematically ones.

![Fig. 1. Generalized kinematic scheme of a serial-parallel manipulator.](image)

All joints are modelled as kinematic joints with different number of restrictions, which give 6 DoF for each drive chain. In this way the number DoF of the structure is defined by the number DoF of the serial chain h. Let generalized parameters are accepted to be the parameters of the relative motions in all joints - elastic and non-elastic of the structure, presented by (k x 1) vector

\[ \Theta = [qq₁]^T \]

where

\[ q = [q₁, …, qₜ]^T \]

is an (h x 1) vector of the generalized coordinates in the joints of the main serial chain with h DoF and

\[ q^T = [w;1]^T \]

is a (6m x 1) vector of coordinates in the joints of the actuator chains. Above

\[ w = [w₁,..., wₘ]^T \]

is an (5m x 1) vector of coordinates in the passive joints of the actuator chains, and

\[ l = [l₁,..., lₘ]^T \]

is an (m x 1) vector of coordinates in the motor linear joints of the actuator chains.
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Let the Cartesian coordinates of the end effector $M$ are denoted as

\[(2.6) \quad X = [X_1, ..., X_n]^T, \nu \leq 6.\]

The relation between the parameters of the basic serial chain (2.2) and the parameters of the end effector (2.6) is known as a direct problem of the kinematics of the serial chain. This problem on the level of displacements and velocities is presented by the equations $X = \Psi(q)$ and

\[(2.7) \quad X = Jq\]

where $J = \left[\partial X / \partial q\right]$ is the $(\nu \times h)$ matrix of Jacobi.

In the parallel structure each closed loop implies the appearance of a connection between the generalized parameters (2.1). These connections are expressed by $6m$ scalar functions for the structure including $m$ parallel loops: $\Psi_i(\theta) = 0, i = 1, ..., 6m$. The differentiation of above equations gives the relation

\[(2.8) \quad H_q \frac{dq}{dt} + H_w \frac{dw}{dt} + H_l \frac{dl}{dt} = 0\]

The matrix of partial derivations $H_q$, $H_w$, and $H_l$ with size $(6m \times h)$, $(6m \times 5m)$ and $(6m \times m)$ allows to produce the summarized matrix of the partial derivatives

\[(2.9) \quad D = \left[\begin{array}{c} \frac{\partial q}{\partial q} ; \frac{\partial w}{\partial q} ; \frac{\partial l}{\partial q} \end{array} \right]^T = [E; W; L]^T\]

where $E$ is unitary $(h \times h)$ matrix, $W$ is a $(5m \times h)$ matrix and $L$ is a $(m \times h)$ matrix, or $D = [E; H]^T$. According to (2.8) we can reduce the $(6m \times h)$ matrix

\[(2.10) \quad H = [W; L]^T = \frac{\partial q}{\partial q}^{-1} = -[H_w; H_l]^{-1} H_q\]

where $[H_w; H_l]$ is a $(6m \times 6m)$ invertible matrix.

Using matrix (10) we have the relations between generalized velocities:

\[(2.11) \quad \dot{q}_i = Hq,\]
\[(2.12) \quad \dot{w} = Wq,\]
\[(2.13) \quad \dot{l} = Lq.\]

The above equations allow determining the velocities $\dot{q}_i$ with dimension $6m$ as a function of the generalized velocities $q$ with dimension equal to the DoF $h$ of the structure.

When the number of parameters (5) is equal to the DoF $m = h$, these parameters can be selected as independent parameters. In relations (2.13) $L$ is a $(h \times h)$ matrix and inverse relation is possible:

\[(2.14) \quad q = L^{-1} l.\]

Equations (2.7) and (2.14) allow determining the velocities of end-effector, while equations (2.11) and (2.14) - the velocities of passive joints, as function of velocities of linear actuator joins1:

\[(2.15) \quad X = JL^{-1} l\]
(2.16) \( q_1 = HL^{-1}l \).

By micromanipulations the above equations give the relations between small motions of microactuators \( \Delta l \), small motions of the end-effector \( \Delta X \) and small motions in passive joints \( \Delta q_l: \Delta X = JL^{-1} \Delta l \) and \( \Delta q_1 = HL^{-1} \Delta l \).


Denote by \( \mathbf{P} = [P_1, ..., P_v]^T \) the \((v \times 1)\) vector of the external forces and torques applied to the end-effector, corresponding to Cartesian coordinates (2.6). Denote by \( \mathbf{Q} = [Q_1, ..., Q_h]^T \) the \((h \times 1)\) vector of the generalized forces and torques in the joints of the main chain corresponding to the general coordinates (2.2). According to the principle of virtual work and equation (2.7), the connection between forces \( \mathbf{P} \) and \( \mathbf{Q} \) is as follows:

\[
(3.17) \quad \mathbf{Q} = J^T \mathbf{P}
\]

Denote by \( \mathbf{F}_q = [F_{q_1}, ..., F_{q_h}]^T \) and \( \mathbf{F}_w = [F_{w1}, ..., F_{w5m}]^T \) \((h \times 1)\) and \((5m \times 1)\) vectors of the forces and torques in the elastic joints, corresponding to coordinates (2.2) and (2.4). Denote by \( \mathbf{F}_l = [F_{l1}, ..., F_{l_m}]^T \) the \((m \times 1)\) vector of the driving forces in the linear joints correspond to the coordinates (2.5). Above vectors can be summarized in the \((h + 6m) \times 1\) vector of forces and torques, corresponding to the coordinates (2.1) \( \mathbf{F} = [\mathbf{F}_q; \mathbf{F}_w; \mathbf{F}_l]^T \).

According to the principle of virtual work and the equation (2.12), (2.13) the relation between forces \( \mathbf{F} \) and \( \mathbf{Q} \), using summarized matrix (2.9), is as follows:

\[
(3.18) \quad \mathbf{Q} = \mathbf{D}^T \mathbf{F}
\]
\[
(3.19) \quad \mathbf{Q} = \mathbf{F}_q + \mathbf{W}^T \mathbf{F}_w + \mathbf{L}^T \mathbf{F}_l
\]

Equations (17) and (19) produce

\[
(3.20) \quad J^T \mathbf{P} = \mathbf{F}_q + \mathbf{W}^T \mathbf{F}_w + \mathbf{L}^T \mathbf{F}_l
\]

Differentiation of above equation with respect to parameters (2.2) and neglect the second partial derivatives, gives

\[
(3.21) \quad J^T \frac{\partial \mathbf{P}}{\partial \mathbf{X}} J = \frac{\partial \mathbf{F}_q}{\partial \mathbf{q}} + \mathbf{W}^T \frac{\partial \mathbf{F}_w}{\partial \mathbf{w}} + \mathbf{L}^T \frac{\partial \mathbf{F}_l}{\partial \mathbf{l}} \mathbf{L}
\]

Considering micromanipulator structure as a system with concentrated compliance in the joints gives

\[
(3.22) \quad \mathbf{K} = J^{-T} [\mathbf{K}_q + \mathbf{W}^T \mathbf{K}_w \mathbf{W} + \mathbf{L}^T \mathbf{K}_l \mathbf{L}] J^{-1}
\]

where \( \mathbf{K} = \partial \mathbf{P} / \partial \mathbf{X} \) is \((v \times v)\) matrix of the Cartesian stiffness of the end effector; \( \mathbf{K}_q = \partial \mathbf{F}_q / \partial \mathbf{q} \) is diagonal \((h \times h)\) matrix of the shaft stiffness in the joints of the main serial chain; \( \mathbf{K}_w = \partial \mathbf{F}_w / \partial \mathbf{w} \) is diagonal \((5m \times 5m)\) matrix of the shaft stiffness in the passive joints of the driving chains; \( \mathbf{K}_l = \partial \mathbf{F}_l / \partial \mathbf{l} \) is diagonal \((m \times m)\) matrix of the shaft stiffness in the driving joints.

A preliminary tensioning of the mechanical micromanipulation system is necessary in order to eliminate the backlash and to improve the performance of the piezo-actuators.

The following two approaches can be used for tensioning of the manipulator:
- deflection from the initial manipulator state by \( m = h \) driving joints motion introduced in the assembly;
- preliminary tensioning of the separate elastic joints with number \( j \), \( j \geq h \).

4.1. Tensioning by means of deflection from the initial state

This can be achieved by means of an assembly deflection \( \delta l \) in the driving joints, which leads to deflection in all the system joints according to (2.14), (2.12) and deflection of the end-effector according to (2.7) defined by the equations:

\[
\begin{align*}
\delta q &= L^{-1} \delta l \\
\delta w &= W \delta q = WL^{-1} \delta l \\
\delta X &= J \delta q = JL^{-1} \delta l
\end{align*}
\]

These deflections lead to elastic joints forces defined by the equations:

\[
\begin{align*}
F_q &= k_q \delta q = k_q L^{-1} \delta l \\
F_w &= k_w \delta w = k_w WL^{-1} \delta l
\end{align*}
\]

where \( k_q \) and \( k_w \) are stiffness matrices of the passive joints of the basic serial chain and of the driving chains, respectively. The tensioned elastic system according to (3.19) is in a static equilibrium:

\[
Q = F_q + W^T F_w + L^T F_l = 0
\]

The diagonal matrices \( k_q \) and \( k_w \) contain non-zero components, responding to elastic joints and zero components responding to kinematic joints. The number of non-zero components \( j \) must be bigger or equal to the DoF \( j \geq h \) in order to achieve full degree of tension of all the actuators and limbs within the system. Equation (4.28) allows definition of the forces of the driving joints \( F_l \) in number \( h \) as a function of the forces \( F_q \), \( F_w \) in number \( j \geq h \):

\[
F_l = -L^T [F_q + W^T F_w]
\]

4.2. Tensioning by Deformations in the Elastic Joints

In the manipulator structure with \( m \) driving joints there are \( j \) passive joints, which can be elastic. Because the driving joints \( m = h \), by means of which the piezo-actuators are modelled are hundreds of times more rigid then the elastic manipulator joints, it is accepted that the system has 0 DoF. The tensioning of the elastic joints does not lead to a change in the manipulator position, but only in a change of the internal forces. For the actuator tensioning, the number of the elastic joints \( j \) must be bigger than the number of the DoF \( j \geq h \). The preliminary joint deformations can be defined by the vectors:

\[
\delta q^0 = [\delta q^0_1, \ldots, \delta q^0_h]^T
\]
(4.31) \[ \delta w^0 = [\delta w_1^0, ..., \delta w_{5m}^0]^T, \]
where the components of which corresponding to non-elastic (kinematic) joints are equal to 0.

The joint stiffnesses are represented by the diagonal matrices \( k_q \) and \( k_w \), which contain non-zero components corresponding to the elastic joints and zero components connected to the kinematic joints.

The preliminary deflections lead to appearance of forces in the elastic joints defined by equalities:

\[
\begin{align*}
F_q &= k_q \delta q^0 \\
F_w &= k_w \delta w^0.
\end{align*}
\]

The driving elastic joints forces are in a static equilibrium (3.19). The equation (4.28) defines the links among all the joint forces and allows according (4.29) the derivation of the driving joints forces in number \( h \) as a function of the elastic forces in number \( j \geq h \)

5. Conclusion.
Piezo actuated micromanipulators with serial-parallel structure including elastic joints are subject of this paper. A kinematic model of the micro manipulators is build using a pseudo rigid body method, where elastic joints are modelled as revolute joints. A stiffness model is created to estimate the manipulator stiffness by stiffness reduction of all elastic joints. Two approaches are proposed and presented here for preliminary tension of parallel manipulator structure: - Deflection from the initial manipulator state by driving joints motion implemented in the assembly; - Preliminary tensioning of separate elastic joints.

REFERENCES