ABSTRACT. The elastic-plastic behaviour of rapidly solidified Al based (FeSi) - enriched alloys is considered. Due to fast cooling the material can be regarded as “natural” (in situ) composite, containing intermetallic and non-intermetallic compounds of different shapes, mechanical properties and volume fractions. A new multilevel mechanical model for the "in situ" composite is proposed considering the aluminium matrix as a micropolar elastic plastic Cosserat material and the hardening phases as pure elastic ones. A two steps homogenization procedure is applied to obtain the overall properties of multiphase “in situ” composite. A variational approach is applied to evaluate the equivalent stress on macro level at the transition from micro to macro scale. The model is developed using information provided by microstructural investigations and EDX analysis. The multistage modelling of bulk material manufacturing process is simulated using the FEM. The model is implemented as user defined subroutine into the FE code MARC. The influence of the microstructural size parameters on the hardening behavior is discussed.

KEY WORDS: multiphase composites, multiscale modeling, rapidly solidified Al alloys

1. Introduction

The elastic-plastic behaviour of Rapidly Solidified (RS) AlFe9V1Si2 alloys is considered. The initial material is powder, subjected to densification, leading to bulk material. Each granule of the powder consists of Al-solid solution, as a matrix and of intermetallic precipitations, as a strengthening phase. These alloys can be
regarded as multiphase “in situ” composites. The scientific interest to these alloys is based on their thermal stability up to 400 °C, light weight combined with high mechanical strength. The RS technique ensures a fine microstructure leading to high mechanical properties. Microstructure observations [1] show that the intermetallic phase consists of very fine almost spherical inclusions with different diameters, randomly distributed in the matrix. The precipitations are divided into three groups, depending on its diameter. The multilevel mechanical model proposed in [2] for two-phase composites, is developed here for multiphase materials.

2. Modelling

The modelling is provided in two levels. On micro level the Al alloy considered is a multiphase particle reinforced composite. The ductile matrix is modeled as micropolar elastic plastic work-hardening material. The quaternary phase precipitations ($\text{Al}_{12}\text{Fe}_5\text{Si}_3\text{V}_2$) which are much harder than the matrix, are assumed elastic, sphere-shaped with different diameters. They can be grouped in a finite number $n > 2$ of sets, according to their average diameters $d_i, i = 1, \ldots, n$. At this level a two-step homogenization procedure is applied. After this homogenization we deal on macro level with a mono-phase density dependent material, subjected to closed die cold compaction. According to the pseudograins model [3] the multiphase Representative Volume Element (RVE) with volume $V$, consisting of matrix and $n$ phases, is equivalent to a RVE, consisting of $n$ two-phase pseudograins. The volume $\Omega_i$ of each pseudograin contains the volume $V_i$ of phase $i$ with inclusions diameter $d_i$ and part of the matrix phase with volume $V_{0i}$, so that the following relations hold:

\[
\frac{V_i}{V_j} = \frac{V_{0i}}{V_{0j}} = \sum_{i=1}^{n} C_i = C_{\text{sum}} = 1 - C_0 = \frac{V_i}{\Omega_i} = \frac{V_j}{\Omega_j},
\]

where

\[
V_0 = \sum_{i=1}^{n} V_i, \quad V_j = \sum_{i=1}^{n} \Omega_i, \quad V_{0i} = \sum_{i=1}^{n} V_{0i}, \quad C_i = \frac{V_i}{V}, \quad C_0 = \frac{V_0}{V}, \quad C_j = \frac{\Omega_j}{V}.
\]

In the above relations $C$ means volume fraction, the subscript 0 stays for the matrix and the subscript $i$ - for the phase $i$.

In each pseudograin the matrix is modeled as elastic plastic Cosserat continuum with internal length parameter $l_m$, shear modulus $G_0$, bulk modulus $K_0$, and micropolar moduli $\beta, \gamma, \kappa$. The intermetallic precipitations are assumed elastic with shear moduli $G_i$ and bulk moduli $K_i, (i = 1, \ldots, n)$.

For each pseudograin updated Mori-Tanaka homogenization (type I) [2] is applied and overall elastic plastic moduli are obtained in the form:
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(2.2) \[ K^s_{\alpha} = K_0 \left[ 1 + \frac{C_{\text{sum}}(K_i - K_0)}{C_0 a_0^s (K_i - K_0) + K_0} \right], G^s_{\alpha} = G_0 \left[ 1 + \frac{C_{\text{sum}}(G_i - G_0^s)}{C_0 b_0^s (G_i - G_0^s) + G_0^s} \right] \]

where upper script \( s \) means secant moduli at plastic state. In the case of pure elastic behaviour the upper scripts \( s \) are omitted elsewhere in the paper. In (2.2) the following notations are used:

(2.3) \[ a_0^s = \frac{3K_0}{3K_0 + 4G_0^s}, \quad b_0^s = \frac{6(K_0 + 2G_0^s)}{5(3K_0 + 4G_0^s)} - \frac{6k^s}{5(k^s + G_0^s)} R_i(\eta_i), \]

\[ R_i(\eta_i) = e^{-\eta_i} \left( \eta_i^2 + \eta_i^3 \right) (\eta_i ch\eta_i - sh\eta_i), \quad \eta_i = \frac{d_i}{2h}, \quad h = \left[ \frac{(G_0^s + k^s)(\gamma^s + \beta^s)}{4G_0^s k^s} \right]^{1/2} \]

After the first homogenization step the material of each pseudograin is Cauchy elastic plastic. Further the conglomerate of \( n \) pseudograins is subjected to a second homogenization (type II), according to self-consistent theory [4]. The final “in situ” composite moduli \( K^s, G^s \) are defined by the following non linear equations:

(2.4) \[ \sum_{i=1}^{n} \frac{\tilde{C}_i}{1 - \frac{3K^s_i}{3K^s + 4G^s} \left( 1 - \frac{K^s_0}{K^s} \right)} - 1 = 0, \quad \sum_{i=1}^{n} \frac{\tilde{C}_i}{1 - \frac{2(3K^s_i + 6G^s_i)}{5(3K^s + 4G^s)} \left( 1 - \frac{G^s_0}{G^s} \right)} - 1 = 0 \]

In (2.4) \( K^s, G^s \) are secant bulk and shear moduli, respectively. For describing the transition from elastic to plastic state we assume the following equivalent stress:

(2.5) \[ \sigma^s = \frac{3}{2} \sigma^{(i)} \sigma^{(j)} + \frac{3}{2I_m} m^{(i)} m^{(j)} + \frac{3}{2I_m} m^{(i)} m^{(j)} \]

and corresponding yield condition for the matrix on micro level:

(2.6) \[ \sigma_e = \sigma_{\text{pAl}}(\bar{\sigma}_p) = \sigma^{0}_{\text{pAl}} + h_0 \bar{\sigma}_p \]

where \( \sigma_{\text{pAl}} \) is the yield stress of the Al based matrix, \( \sigma^{0}_{\text{pAl}} \) is its initial value, \( \bar{\sigma}_p \) is the equivalent plastic strain on micro level and \( h_0, m \) are matrix hardening parameters. Symbols \( (\ldots) \) and \( \langle \ldots \rangle \) in the subscript denote the symmetric and anti-symmetric parts of a tensor, \( (\ldots)^\prime \) means deviator. We assume that the plastic stage of the composite is reached if the yield condition for the matrix material is satisfied, as the inclusions are pure elastic and do not undergo plastic deformation. We propose the following yield condition on macro level:

(2.7) \[ \langle \sigma^s_e \rangle_0 = \sigma^2_{\text{pAl}}(\langle \bar{\sigma}_p \rangle_0) \]

where \( \langle \ldots \rangle_0 \) means averaged value over the matrix domain in RVE.
The modelling on macro level requires two different relations to be established:

- between the averaged over the matrix equivalent stress $\langle \sigma_e^2 \rangle_0$ and the stress invariants on macro level $\frac{1}{2} \Sigma_i \Sigma'_i$ and $\Sigma_{kk}$:
- between the averaged over the matrix equivalent plastic strain $\langle \varepsilon'_p \rangle_0$ and the equivalent plastic strain on macro level $\bar{E}_p$.

The variation technique of Hu et al. [5] is used to evaluate the averaged equivalent stress:

$$\langle \sigma_e^2 \rangle_0 = \frac{1}{C_0} \left[ \frac{3}{2} \Sigma_i \Sigma'_i + \frac{1}{l^2_m G^{s2}} \frac{\partial \bar{G}^s}{\partial \beta^s} + \frac{1}{l^2_m G^{s2}} \frac{\partial \bar{G}^s}{\partial \gamma^s} \right]$$

The derivatives, appearing in (2.8) are calculated from (2.2) and (2.4).

For the second relationship we assume:

$$\langle \varepsilon'_p \rangle_{\text{RVE}} = \langle \varepsilon'_p \rangle_0 = \bar{E}_p / \sqrt{C_0}$$

Finally the yield condition of the “in situ” composite takes the form:

$$F = \frac{3}{2} \Sigma_i \Sigma'_i + \frac{A^2}{9B_c} \Sigma_{kk} - \sigma_{pc}^2 = 0,$$

where

$$\frac{1}{A^2} = \frac{1}{C_0} \left( \frac{G^{s2}}{G^{s2}} \frac{\partial \bar{G}^s}{\partial \beta^s} + \frac{l^2_m G^{s2}}{l^2_m G^{s2}} \frac{\partial \bar{G}^s}{\partial \gamma^s} \right)$$

and

$$\sigma_{pc} = A_c \sigma_{pAl}(\langle \varepsilon'_p \rangle_0) = A_c \left[ \sigma_{pAl}^0 + h_t (\bar{E}_p / \sqrt{C_0})^m \right]$$

is the composite yield limit. The associated flow rule is assumed. It is well known that the von Mises yield criterion is good enough to describe the inelastic behaviour of aluminium alloys. If we assume that the homogenized composite is plastically incompressible, the coefficient $A_c / B_c$ in (2.10) should vanish. Then the requirement $1 / B_c = 0$ has to be fulfilled, as $A_c \neq 0$. The coefficient connected with the first stress invariant on macro level is equal to zero only if $K_0 = K_\mu$. This means that the elastic volume change of the matrix is equal to the volume change of the inclusions.
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Having in mind that we deal with a naturally arisen composite, such a restriction seems to be reasonable. Experiments provided on bulk samples made of rapidly solidified AlFe9V1Si2 alloy subjected to compression showed no debonding between the matrix and the precipitations and they are in maintenance of this assumption. If the bulk moduli of the composite compounds are equal, the elastic properties of the intermetallic phases vary in the narrow interval $E_0 < E_i < E_0/(1-\nu_0)$. Typical hardening curves corresponding to relation (2.12) for the homogenized “in situ” composite with three sets of inclusions are illustrated in Fig. 1.

3. Numerical simulations and results

Processing of the bulk material from rapidly solidified powder of AlFe9V1Si2 alloy during cold closed die compaction of cylindrical billets is simulated using the Finite Element (FE) code MARC with included model dependent user subroutines. Four-noded axisymmetric quadrilateral isoparametric FE are applied. The Al-based matrix parameters are: $E_0 = 71 \text{ GPa}$, $\nu_0 = 0.34$, $\kappa = 53 \text{ GPa}$, $\beta = l_m^2 C_0$, $\gamma = l_m^2 \kappa$, intrinsic length $l_m = 4.5 \mu m$, initial yield stress $\sigma_{p0} = 122 \text{ MPa}$. The microstructural observations showed that the intermetallic precipitations can be divided into three sets, depending on their size as follows: $d_1/l_m = 0.47$, $d_2/l_m = 1.11$, $d_3/l_m = 0.72$ with volume fractions: $C_1 = 0.032$, $C_2 = 0.070$ and $C_3 = 0.073$. The elastic moduli of the inclusions are $K_i = K_0$, $E_i/E_0 = 2.2$, $i = 1, 2, 3$. The numerical results were compared with experiments performed on compacted cylinders of preconsolidated powders of fraction $+125–180 \mu m$, see Fig. 2. The experimental points are obtained on the
base of the Brinell hardness measurements at different height reductions of the specimens, using the empirical relationship \( \sigma_{pw}^{\text{exp}} = \frac{\text{HB}}{3} \), where \( \text{HB} \) is in MPa.

The results of numerical simulations show that the two structural level approach and two steps homogenization of multiphase metal matrix composites enable the size effects on micro level to be elicited on macro level.

REFERENCES