TRIAXIAL STATE OF STRESS AT CRACK TIPS STUDIED BY CAUSTICS

E.E. GDOUTOS
Office of Theoretical and Applied Mechanics of the Academy of Athens
School of Engineering, Democritus University of Thrace
GR-671 00 Xanthi, Greece
e-mail: egdoutos@civil.duth.gr

ABSTRACT. The optical method of caustics is used to study the triaxiality of the stress field in the neighborhood of a crack tip in a tension specimen. It is found that the state of stress in the vicinity of the crack tip is three-dimensional up to a limiting distance from the tip, after which it approaches to plane stress. This distance depends mainly on the ratio of the crack length to specimen thickness.

KEY WORDS: Stress triaxiality, stress intensity factors, cracks, caustics, plane stress, plane strain

1. Introduction

The optical method of caustics is sensitive to stress gradients, and it has extensively been used for the solution of crack problems [1-6]. According to the method the stress singularity at the crack tip is transformed to an optical singularity. A highly illuminated curve, so-called caustic, is obtained on a viewing screen at a distance from the crack tip. The dimensions of the caustic are related to the state of stress near the crack tip. Thus the stress intensity factor which governs the stress field can be determined by measuring characteristic dimensions of the caustic, usually, its diameter perpendicular to the crack. In the present paper the method of caustics is used for the investigation of the triaxiality of the state of stress in the neighborhood of a crack tip.

2. The Optical Method of Caustics

In the optical method of caustics a specimen is illuminated by a light beam and the reflected or transmitted rays undergo a change of their optical path dictated by the stress field. The change of the optical path is caused by the variation of the thickness and refractive index as the specimen is loaded. At stress gradients resulting at crack tips, the reflected or transmitted rays generate a highly illuminated three-dimensional surface in space. When this surface is intersected by a reference screen,
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a bright curve, the so-called caustic curve, is formed. For transparent materials three caustics are formed by the light rays reflected from the front and rear surfaces and those transmitted through the specimen. For opaque materials, only one caustic is formed by the reflected light rays from the front surface of the specimen. The dimensions of the caustic are related to the state of stress near the crack tip. An optical arrangement for the method of caustics is shown in Fig. 1. For the case of a mode-I through-the-thickness crack the stress intensity factor $K_{\text{exp}}$ is given by [1]

$$K_{\text{exp}} = 0.0934 \frac{D^{5/2}}{z_0 ct m^{3/2}}$$

where $z_0$ is the distance between the specimen and the viewing screen where the caustic is formed, $c$ is the stress optical constant of the specimen under conditions of plane stress, $t$ is the specimen thickness, $m$ is the magnification factor of the optical arrangement defined as the ratio of a length on the reference screen where the caustic is formed divided by the corresponding length on the specimen and $D$ is the transverse diameter of the caustic at the crack tip (Fig. 2). The above equation is valid when the state of stress in the vicinity of the crack tip is plane stress.

The caustic is created by the light rays reflected from the circumference of a circle, the so-called initial curve, which surrounds the crack tip. The radius of the initial curve is given by

$$r = 0.316 D$$

![Fig. 1. Optical setup of the method of caustics](image)
3. Experimental
Specimens made of Plexiglas, of thickness $t = 3.0, 4.5, 9.5$ and $12.5$ mm and width $w = 42.4, 47.5, 51.5$ and $63.5$ mm, with an edge notch of length $a = 15.5$ mm were subjected to a progressively increasing tensile loading in an Instron testing machine. The specimens were illuminated by a convergent, divergent or parallel monochromatic light beam produced by a Ne-He laser. The caustic curves obtained from the light rays reflected from the front or rear faces of the specimen, or those transmitted through the specimen were recorded on a viewing screen placed at a distance $z_0$ from the specimen. Caustics were obtained at different load levels for various values of the magnification factor of the optical arrangement and the distance $z_0$. In this way, a host of caustics were obtained from different distances $r$ of the initial curve from the crack tip.

4. Results
Experimental values of stress intensity factor, $K_{\text{exp}}$, were obtained. These values were compared with theoretical values of stress intensity factor $K_{\text{th}}$ given by [7]
Note that the experimental values of stress intensity factor are obtained under the assumption that the initial curve of the caustic lies in the region near the crack tip where plane stress conditions dominate. Thus, if the values of $K_{\text{exp}}$ and $K_{\text{th}}$ coincide, this means that the initial curve of the caustic lies in the region where the state of stress is plane stress. In case the values of $K_{\text{exp}}$ and $K_{\text{th}}$ do not coincide, this implies that the initial curve lies in the region where the state of stress is three-dimensional.

In order to characterize the three-dimensionality of the stress field near the crack tip we introduce an empirical triaxiality factor $k$, such that

$$
\sigma_z = k \nu (\sigma_x + \sigma_y)
$$

where $\sigma_z$ is the normal stress perpendicular to the plane of the specimen, and $\sigma_x$ and $\sigma_y$ are the in-plane stresses. $k$ takes the values of 0 and 1 for plane stress ($\sigma_z = 0$) and plane strain ($\sigma_z = \nu (\sigma_x + \sigma_y)$), respectively. $k$ can be determined experimentally by comparing the values of $K_{\text{exp}}$ and $K_{\text{th}}$.

Fig. 3 presents the variation of $K_{\text{exp}}/K_{\text{th}}$ versus $r/t$ for $a = 15.5$ mm, $t = 4.5$ mm and $w = 47.5$ mm (a) and $w = 63.5$ mm (b). Points in figures correspond to different values of the applied load, $P$, the magnification factor of the optical arrangement, $m$, the distance between the specimen and the viewing screen where the caustic is formed, $z_0$. Note from figure that the ratio $K_{\text{exp}}/K_{\text{th}}$ increases with $r/t$ and reaches a plateau value equal to one as the radius of the initial curve takes a limiting value $r_c$. At that value of $r = r_c$ the state of stress in the neighborhood of the crack tip becomes plane stress. For distances $r$ smaller than $r_c$ the state of stress is three-dimensional, while for values of $r$ larger than $r_c$ plane stress conditions dominate. From Fig 3 the value of $r_c$ for which $K_{\text{exp}}/K_{\text{th}}$ can be determined. Form this figure and analogous figures we obtained that $r_c = t$ for $t = 3$ mm, $r_c = 0.9t$ for $t = 4.5$ mm, $r_c = 0.5t$ for $t = 9.5$ mm and $r_c = 0.4t$ for $t = 12.5$ mm. Thus the critical value of $r$ for which the state of stress becomes plane stress depends not only on $t$, but also on the geometrical characteristics of the cracked plate, especially the ratio of the crack length to specimen thickness.
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Fig. 3. Variation of $K_{\text{exp}}/K_{\text{th}}$ versus $r/t$ for $a = 15.5$ mm, $t = 4.5$ mm and $w = 47.5$ mm (a) and $w = 63.5$ mm (b)

5. Conclusions
An investigation was performed on the state of stress in the neighborhood of the crack tip. Results were obtained from edge-notched specimens under tension by using the optical method of caustics. It is obtained that the state of stress in the neighborhood of a crack tip is three-dimensional up to a limiting value, after which it becomes plane stress. This critical distance depends on the ratio of crack length to specimen thickness, $t$, and varies from $t$ to $0.4t$ for plate thicknesses from 3 to 12.5 mm. Very close to the crack tip the state of stress is plane strain. The three-dimensionality of the state of stress near the crack tip can be characterized by an empirical triaxiality factor, which can be determined by the method of caustics

REFERENCES


