ABSTRACT. Two phase flows with variable density are the flows which as a result of some heat and mass transfer processes change their temperature respectively density. As an example could be shown the drying process - the moisture in the particle goes to the carrying gas media because of the realized heat and mass transfer between phases. The process continues up to reaching the certain equilibrium humidity in the material. As a result of the moisture evaporation the particle changes your own density and mass respectively.

A modification of the known turbulent $k-\varepsilon$ model is done. It includes two equations for turbulent kinetic energy (gas phase and admixture phase). Also in these two equations an extra term giving the mutual impact of turbulent energy of one to another phase is written. Numerically are investigated the velocity fields of the two phases as well as the change of some basic parameters of the flow (temperature, density, mass) by varying with initial temperatures and velocities of two phases also and the initial particle diameter.

KEY WORDS: two-phase flows; turbulent model; non-isothermal flow; variable density.

1. Introduction

Two phase flows with variable density are the flows where as a result of heat and mass transfer processes the temperature, respectively the density of the phases is changing. Such type fluid flows have wide engineering application. As an example
here could be mentioned the drying processes. The moisture in the particle goes to
the carrying media (gas phase) because of the realized heat and mass transfer
processes. The process continues up to reaching the certain equilibrium humidity into
the material. As a result of moisture evaporation process the particle changes your
own density and mass respectively.

The mathematical model was work out using the two-fluid flow scheme. Each of the
phases is reviewed as a separate fluid media with your own velocity, temperature,
and density. The admixture phase here is accepted to be an “incompact fluid media”,
and the equation for ideal gas law here could not be used because of the absence
of the stress tensor. The mathematical model includes the following equations:
continuity equation; admixture concentration save; movement; energy transportation.
For each of the phases these equations should be written separately. The connection
between equations is appearing the forces from inter-phase interaction. The density
change of the gas phase is presented by the Clapeyron’s equation, and separate
equation is accepted for the density change of the particles.

Modification of the known $k-\varepsilon$ model here is used for closing the system for
differential equations. The model includes two equations for the turbulent kinetic
energy and one for the dissipation speed.

The complete system of partial differential equations is solved using the finite
difference method with “Dufort-Frankel” discretization scheme. A module in Pascal
is created realizing the numerical solution of the system equation.

Numerically are investigated the velocity fields of the two phases as well as the
change pf some basic parameters of the flow (temperature, density) by varying with
the initial temperatures and velocities of two phases also the initial particle diameter.

2. Mathematical model

Below are presented the continuity equations and the Reynolds’s equations
in two dimensional formulation.

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ y^j U_g \rho_{g} \right] &+ \frac{\partial}{\partial y} \left[ y^j V_g \rho_{g} \right] = 0 \\
\frac{\partial}{\partial x} \left[ y^j U_p \rho_{p} \right] &+ \frac{\partial}{\partial y} \left[ y^j V_p \rho_{p} \right] = 0 \\
\left[ y^j U_p \right] \frac{\partial y}{\partial x} &+ \left[ y^j V_p \right] \frac{\partial y}{\partial y} = \frac{\partial}{\partial y} \left[ y^j \chi V'_{p} \right] - \chi V'_{p} \\
\left[ y^j \rho_g U_g \right] \frac{\partial U_g}{\partial x} &+ \left[ y^j \rho_g V_g \right] \frac{\partial U_g}{\partial y} = \frac{\partial}{\partial y} \left[ y^j \rho_p U_p' V'_{p} \right] - F_{x,y} \\
\left[ y^j \rho_p U_p \right] \frac{\partial U_p}{\partial x} &+ \left[ y^j \left( \rho_p V_p + \rho_p' V'_{p} \right) \right] \frac{\partial U_p}{\partial y} = \frac{\partial}{\partial y} \left[ y^j \rho_p U_p' V'_{p} \right] + F_{x,y} \\
P &= \rho_g RT_g
\end{align*}
\]
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\[ \frac{h_{\text{ij}}}{\rho_{\text{ij}} T_{\text{cp,d}}} \frac{\partial T}{\partial t} = \frac{1}{T_{\text{z}} - T_{\text{p}}} \frac{\partial T}{\partial t} \]

where the equations (2.1-2.2) are the continuity equations for both phases; Equation 2.3 is concentration admixture save; Equations 2.4 and 2.5 are movement equations for both phases; Equation 2.6 is Clapeyron’s equation; Equation 2.7 is admixture phase density change equation.

2.1. Heat transfer modeling

To the system differential equations 2.1÷2.7 is added the equations represent the heat transfer not only between two phases but also the heat transfer between each phase and surrounding media:

\[ -Q \rho \frac{\partial \rho}{\partial t} + F_x y_j \left( U_{\text{g}} - U_{\text{p}} \right) + F_y y_j \left( V_{\text{g}} - V_{\text{p}} \right) - \sum_{i=1}^{3} F_i V_{\text{pi}} + 2 \pi R_j \rho \frac{\partial T_j}{\partial y} \]

\[ \left[ y_j \rho U_{\text{g}} \frac{\partial h_{\text{g}}}{\partial x} + y_j \rho V_{\text{g}} \frac{\partial h_{\text{g}}}{\partial y} \right] - \left[ y_j \rho \rho h' \frac{\partial V_{\text{g}}}{\partial y} \right] \frac{\partial U_{\text{g}}}{\partial y} \]

Where:

\[ \rho = \frac{\partial T_{\text{g}}}{\partial y} \frac{\partial T_{\text{p}}}{\partial y} \]

To the system equations 2.1÷2.9 should be also added the equation for concentration conservation:

\[ \left[ y_j \left( \frac{\partial \rho}{\partial t} \right) \frac{\partial h_{\text{g}}}{\partial x} \right] + \left[ y_j \rho \frac{\partial h_{\text{g}}}{\partial x} \right] - \left[ y_j \rho \rho h' \frac{\partial V_{\text{g}}}{\partial x} \right] \frac{\partial U_{\text{g}}}{\partial x} \]

2.2. Turbulence modeling

Here is used the described in [1] [2] modified \( k_g - k_p - \varepsilon \) model of turbulence. The difference with the standard \( k - \varepsilon \) model is that for each of the phases a separate equation of turbulent kinetic energy is written:
In the up-shown equations (2.11÷2.13) the connection between two phases is given with the concentration.

The turbulent viscosities as well as the dissipation and mixing length for both phases are defining according the expressions:

\[ \nu_g = C_v \cdot K_g^{0.5} \cdot L; \quad \nu_p = C_v \cdot K_p^{0.5} \cdot L; \quad \varepsilon = C_D K_g^{0.5} \cdot L; \quad L = C_1 (y_{0,1} - y_{0,9}) \]

### 2.3. Boundary conditions

The complete system of differential equations is solving using the following boundary conditions:

- for symmetry axis (y = 0):

\[ \frac{\partial U_g}{\partial y} = \frac{\partial U_p}{\partial y} = 0; \quad \frac{\partial T_g}{\partial y} = \frac{\partial T_p}{\partial y} = 0; \quad \frac{\partial \rho_g}{\partial y} = 0; \quad \frac{\partial \rho_p}{\partial y} = 0; \quad \frac{\partial U_g' V_g'}{\partial y} = 0; \quad \frac{\partial U_p' V_p'}{\partial y} = 0 \]
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- for the boundary surface \(( y = R_u )\):

\[
\begin{align*}
U_g V'_g &= \frac{\partial V'_g \rho'_p}{\partial y} = 0; & U'_p V'_p &= \frac{\partial U'_p V'_p}{\partial y} = 0; & V'_p \rho'_p &= \frac{\partial V'_p \rho'_p}{\partial y} = 0;
\end{align*}
\]

\[
\frac{\partial T_g}{\partial y} = \frac{\partial T_p}{\partial y} = 0; \quad V'_g = V'_p = 0; \quad U'_p = U_2; \quad \rho'_p = 0; \quad T'_g = T'_p = T_2; \quad \rho_g = \rho_2
\]

3. Numerical solution. Results

The system of differential equation is solved by using the finite differences method. The applied scheme here Dufort-Frankel. After discretization the complete system could be presented with the following general characteristic equation:

\[
A \frac{\partial Z}{\partial x} + B \frac{\partial Z}{\partial y} = C \frac{\partial^2 Z}{\partial y^2} + D
\]

where \(Z, A, B, C,\) and \(D\) are the respective variables and quantities.

Below are presented some results from numerical solution. In order to be verified the mathematical model and the program efficiency the results from numerical solutions have been compared with the experimental data.

In figure 1 is shown the comparison between experimental data [3] and the data from numerical solution for velocity change of the particle in longitudinal direction. The numerical solution was made by using \(k - \varepsilon\) and modified \(k_g - k_p - \varepsilon\) model of turbulence. The input data for the numerical experiment are: velocities - 

\(U_g = U_0 = 25 \text{ m/s} ;\) diameter of the particles - \(d_p = 50 \mu\text{m} ;\) temperatures of the phases - \(T_g = T_0 = 293 K ;\) densities of the phases - \(\rho_g = 1.177 \text{ kg/m}^3 ,\) \(\rho_g = 2200 \text{ kg/m}^3 .\) There is a good correlation between experimental data and the data from two numerical solutions (using general and the modified turbulence models) especially up to \(X/Y_0 = 20 .\) After this section the modified turbulent model show better results than the general. This could be explained with the additional terms in the both turbulent kinetic energy equations providing possibilities of mutual interaction between phases. In fig. 2 are presented the results from numerical solution about the temperature distribution. The initial conditions are the same as the previous case. On the figure are shown two basic cases – spreading of hot jet in cool media \((T_g = 353; 453 \text{ and } 653K ),\) and cool jet spreading in the hot media \(T_0 = 273; 283K .\)

The surrounding media temperature is \(T_2 = 293K .\) Independently from the regime the most temperature change is in the initial part where the temperature gradient is significant. For the up-mention regime in the figure 3 is presented the density distribution of the gas phase in horizontal direction. In figure 4 is presented the vertical velocity component distribution. The velocity component in central position has positive values which correspond of mass transportation to the periphery of the
jet, and far away from the jet it has negative values which correspond to mass suction from the surrounding media to the jet.

Fig. 1. Admixture velocity distribution

Fig. 2. Temperature distribution

Fig. 3. Density distribution

Fig. 4. Vertical velocity distribution

4. Conclusion
Modified mathematical model of non-isothermal two phase flow with variable density here is presented. The turbulent model is also modified as two separate equations for turbulent kinetic energy for both phases are here written. The propose system from differential equation is numerically solved. The results from numerical solution are compared with the experimental data.

REFERENCES