A PARAMETRIC ANALYSIS OF SINGLE-FIBRE FRAGMENTATION TEST USING BEM

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ABSTRACT. The single-fibre fragmentation test is one of the most frequently used experimental technique for modeling and studying interfacial properties of composite materials. The purpose of the paper is to show through the use of boundary element analysis the effects of material properties and specimen geometric factors, such as fibre modulus and diameter, interphase modulus and thickness on the interphase stress transfer between the matrix and fibre. The obtained numerical results for the fibre axial stress and the interphase and matrix shear stress distributions are shown in figures and discussed.

KEY WORDS: BEM, Fibre fragmentation test, Interface, Energy release rate

1. Introduction

Single fibre fragmentation test [1] is especially used for characterization of the fibre-matrix interfacial toughness (strength) in composite materials. The aim of the test is to obtain a stable growth of a crack along the fibre-matrix interface in a single fibre sample. Qualitative values of interfacial strength are usually obtained with the use of semi-analytical or numerical solutions obtained after some simplifying assumptions.

Even the geometry of the problem is simple, the numerical analysis of the stress state is not an easy task due to several aspects: the significant differences between the dimensions of the cross section of the sample and the fibre diameter, the singular stresses appearing in the vicinity of the crack tips which require a sharp refinement of the mesh in these regions, the presence of contact in the crack lips, and the presence of residual stresses arisen during the curing of the sample [2-4].

Therefore, a BEM based procedure with multi-domain capability has been implemented to solve elasticity problems with axial symmetry, including the stresses
induced by a uniform change in the strain of the solids. Contact conditions have been taken into account using a weak formulation of the equilibrium and compatibility equations along the interface of the solids, which allows the use of non-conforming meshes in the solids along the contact zone. In order to carry out the numerical analysis, some simplifying assumptions are made: an axial symmetry with respect to the fibre axis, sufficiently long fragments and frictionless contact conditions in the crack faces.

This study has been focused on the case in which there is neither failure at the interface nor in the matrix has been considered first to determine the characteristic lengths of the fragments to be employed in the analysis. The primary solution will be employed to show the behaviour of the singular stresses in the vicinity of the debond crack tip and to compute the energy release rate during crack propagation. The fibre axial stress and interfacial stresses will be computed in the postprocessing stage. The aim of the present paper is to demonstrate that the numerical algorithm employed is suitable for obtaining accurate results for the stress state at the crack tip and to show the solution obtained in the different configurations considered.

2. Description of the single-fibre fragmentation test

Fragmentation samples consist of a sufficiently long fibre embedded in a resin matrix, subjected to tensile load. The tensile load, applied at the ends of the sample, is transformed to the fibre through shear stresses at the fibre-matrix interface. As the fibre presents brittle fracture behaviour, and the maximum allowable strain is much lower in the fibre than in the matrix, after reaching a certain value of the applied load the fibre breaks at its weakest location. After breaking, the axial stress in the fibre decay to zero at the plane of failure, as depicted in Fig. 1(a) [2]. However, the stress transfer through the fibre-matrix interface continues and, beyond a certain distance from the crack, the fibre is still bearing the same axial stress as was prior to the breakage. Therefore, after this first failure occurs, a slight increase on the applied load results in the successive fragmentation of the fibre, as shown in Fig. 1(b). In addition, due to the high amount of energy released, after the fibre breakage takes place, small debond cracks appear in the fibre-matrix interface.

Comparing the growth of these debond cracks with the fragmentation process, three stages can be considered during the completion of the test. In the first stage, fragmentation of the fibre occurs without a noticeable growth of the debond crack. Afterwards, in a second stage, the debond cracks start to grow. This debond growth causes a less efficient shear stress transfer through the interface, which
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consequently induces a decrease in the axial stress in the fibre. Therefore, fragmentation of the fibre continues during this stage, but the new failures at a lower rate in the fibre appear as debond crack grow along the interface. In the final stage, depicted in Fig. 1(c) the debond cracks grow rapidly but the axial stress in the fibre is not high enough to cause the appearance of new fragments. Then an increase on the applied load causes only a further debond growth.

Although the desired behaviour of the sample during the test is the one described above, in some cases, no failure of the interface is obtained during the completion of the test for one of the two following reasons. If the matrix is weaker than the interface, matrix cracks can arise at fragment ends and cause the failure of the sample prior to the debond growth [3]. On the other hand, if the matrix and the interface are tough enough, the second and third stages of the test may not take place. In this case, as the fragmentation process progresses, the amount of fibre which is subjected to high axial stresses diminishes, and consequently, saturation is reached (Fig. 1(d)) when the fragment sizes are two small to allow that the axial stress in the fibre reaches a value high enough to cause new fragments in the fibre.

3. Description of the problem

As was mentioned above, fragmentation samples consist of a sufficiently long fibre embedded in a resin matrix, subjected to tensile load. The tensile load, applied as a uniform strain at the ends of the sample, is transferred to the fibre through shear stresses at the fibre-matrix interface.

The axial fibre stress increases from zero at the ends until the fibre strength is reached. If the loading is continued the repetition of the fragmentation process will occur until all remaining lengths are so short that the shear stress transfer along their lengths can no longer build up enough tensile stress to cause any further failure with increasing strains. This final fragmentation length is referred as the critical length \( l_c \). This fact is usually referred to as saturation. In the case that the only failure in the sample takes place in the fibre, the final average length of the fragments only depends on the fibre failure properties. Based on the theory of shear lag model the interfacial shear strength \( \tau \) can be calculated by a simple force balance\[ \tau_c = \frac{\sigma_f d_f}{2l_c}, \]

where \( \sigma_f, d_f \) are the diameter and a tensile stress of the fibre respectively.

Nevertheless, in most cases, this final average length is not reached because of the appearance of a series of cracks which arise from the ends of fibre fragments and grow through the matrix or the interface.

There are three different crack paths observed in the experiments [3], as shown in Fig. 2: a penny-shape crack and a bi-conical crack, both growing through the matrix, or a bi-cylindrical crack, growing

\[ \begin{align*}
\text{Penny-shape} & \\
\text{Bi-conical} & \\
\text{Debonding} & \\
\end{align*} \]

Fig. 2. Crack patterns
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along the interface, which is often referred to as debonding crack.

The appearance of these cracks implies a relaxation of the axial stress in the fibre, which prevents the appearance of new fibre breakages, and therefore saturation is reached yielding a longer average length of the fragments which depends on the interface or matrix failure properties.

When a debonding crack is obtained, calculation of interfacial shear strength is possible [1, 5] with the use of some assumptions about the stress state in the vicinity of the crack tip and about a crack propagation criterion. If a matrix crack is obtained as a result of the test, there is no possibility of calculating the interface shear strength, although a lower bound for its actual value can be derived from the stress analysis.

3.1. Penny-shape crack

This crack initiates at the end of the fibre crack and grows in radial direction through the matrix in pure Mode I and, therefore, the energy release rate can be calculated using the following integral procedure propose in [3, 6]:

\[ G_I = \lim_{\delta \alpha \to 0} \frac{1}{2} \int_a^{\alpha + \delta \alpha} \sigma_{xy}(x, r_f) \Delta u_y(x - \delta \alpha, r_f) \, dx \]

where \( \Delta u_y(x - \delta \alpha, r_f) = u_y(x - \delta \alpha, r_f) \bigg|_{\text{matrix}} - u_y(x - \delta \alpha, r_f) \bigg|_{\text{fibre}} \) are the components of the relative normal displacement between the matrix and the fibre at the interface in the vicinity of the crack tip.

3.2. Conical crack

In this kind of failure, two crack initiate from the end of each fibre crack and grow with a certain deviation from the axial direction of the sample. The growth of the conical cracks occurs in mixed Mode I and II. Therefore, energy release rate (assuming that the crack grows following a straight path) can be calculated as \( G = G_I + G_{II} \), where \( G_I \) is given in (3.1) and \( G_{II} \) is:

\[ G_{II} = \lim_{\delta \alpha \to 0} \frac{1}{2} \int_a^{\alpha + \delta \alpha} \sigma_{xy}(x, r_f) \Delta u_x(x - \delta \alpha, r_f) \, dx \]

and \( \Delta u_x(x - \delta \alpha, r_f) = u_x(x - \delta \alpha, r_f) \bigg|_{\text{matrix}} - u_x(x - \delta \alpha, r_f) \bigg|_{\text{fibre}} \) are the components of the relative tangential displacement between the matrix and the fibre at the interface in the vicinity of the crack tip.

3.3. Debonding crack

The debonding crack grows along the interface of two materials with different elastic properties. Two different approaches for computing energy release rate in an interface crack have been employed in this study: one assumes that the crack lips are open and the other consists in solving the problem imposing contact conditions between the crack lips.
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If the interface crack faces are supposed to be open, i.e. free of stresses, an oscillatory singular solution is obtained for the normal and tangential stresses ahead of the crack tip and the displacements of the bodies at the crack faces [7]. The energy release rate can be computed as:

$$G = \frac{\mu_m (k_f + 1) + \mu_f (k_m + 1)}{\mu_f \mu_m} \frac{\pi K_k}{8 \cosh^2 (\pi \varepsilon)},$$

where $\mu_f, \mu_m$ are the shear modulus of each material (fibre and matrix),

$$K_k = \lim_{\varepsilon \to 0} \{ \delta \sigma(\partial \alpha, r_f) + \sigma_{\alpha}(\partial \alpha, r_f) \}$$

is the squared modulus of the complex stress intensity factor, $k_{f,m} = 3 - 4\nu_{f,m}$

$$\varepsilon = \frac{1}{2\pi} \log \left( \frac{1 + \beta}{1 - \beta} \right)$$ is a bi-material constant depending on Dundurs’ parameter

$$\beta = \frac{\mu_m (k_f - 1) - \mu_f (k_m - 1)}{\mu_m (k_f + 1) + \mu_f (k_m + 1)}.$$

4. Numerical analysis and results

Taking into account that the dimensions of the sample cross section are much larger than the diameter of the fibre, axial symmetry with respect to the fibre axis has been assumed in the analysis. For this reason, a BEM formulation with axial symmetry has been implemented for the stress analysis of different solids connected by several bonded interfaces or contact regions. Details about the implementation of the numerical method employed can be found in [8, 9].

The main advantage of using BEM is that only the meshing of the boundary of the solids is required for the analysis. As axial symmetry is considered in the study, the analysis domain consists in the radial plane of the sample. Therefore, the boundaries of both solids are constituted by lines, being therefore an easy task to obtain a well suited mesh, even in a problem like this which involves regions with singular stresses in which a steep refinement of the mesh needs to be employed to obtain an accurate solution. Furthermore, BEM has proved to be a powerful tool in the numerical analysis of the stress state in problems involving stress singularities and the possibility of contact between crack faces.

The primary solution of a BEM analysis are the components of the displacements and tractions at the nodes of the boundary mesh, while the components of the displacements of the displacements and stress tensor at any point inside the solid can be computed from this primary solution in a postprocessing stage.

The objective of the first analysis consists in the study of the fragmentation process prior to the appearance of the debond cracks. The geometry and boundary
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conditions for this configuration correspond to half of one fragment with a length of $2L_f$ is sketched in Fig. 3. Results presented in each configuration have been obtained using a fibre radius $r_f = 5 \mu m$ , a sample radius $r_m = 40r_f$ and an applied load $\varepsilon_0 = 3\%$ [10] in all cases and making a parametric variation of fragment length, fibre and interphase modulus, fibre diameter, interphase thickness and crack size. Material properties employed for the analysis are summarized in Table 1. Boundary conditions for all cases are [2]:

- Symmetry conditions with respect to the plane containing the fibre crack.
- Stress free conditions at the external face of the sample.
- Perfect bonding at the interface between fibre and matrix.
- Stress free conditions at the faces of the fibre crack.

Uniform strain $\varepsilon_0$ applied to the sample has been modeled as a constant displacement in the direction of the sample.

Table 1. Mechanical properties of the fibre, interphase and matrix [10]

<table>
<thead>
<tr>
<th></th>
<th>Matrix</th>
<th>Fibre</th>
<th>Interphase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resin</td>
<td>E-glass</td>
<td>Carbon</td>
</tr>
<tr>
<td>$E_m$</td>
<td>2.4 GPa</td>
<td>76 GPa</td>
<td>238.7 GPa</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>0.36</td>
<td>0.22</td>
<td>0.2</td>
</tr>
<tr>
<td>$E_i$</td>
<td>3.75 GPa</td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>

As was stated above, if no cracks appear in the matrix or the interface, fragmentation continues until a critical fragment length is reached, where the fragments are not long enough to transfer, through the interface, an axial load high enough to continue with the splitting of the fibre.

The effect of the successive fragmentations can be observed in Fig. 4, in which axial stress in the middle of the fibre is plotted using three different fragment lengths. In the first configuration ($L_f = 500 \mu m$ ) it can be seen how the axial stress is constant along the majority of the fibre and it decays rapidly towards a zero values in the vicinity of the fibre crack (at $z = 0$ ). If this fragment is broken again at its middle plane a new solution is obtained (corresponding to $L_f = 250 \mu m$ ) in which
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the region where the axial stress in the fibre is constant diminishes significantly. In this region, the fibre is subjected to the same axial stress as was prior to the fragmentation. After a breaking of the new fragments at their middle plane, a new solution are obtained (corresponding to \( L_f = 125 \, \mu m \) ) in which it can be observed that the resulting fragments are not long enough to reproduce the plateau value of the axial stress in the middle of the fragment.

As can be seen, when fragment length is long, at a certain distance from the end of the fragment axial stress reaches its nominal value, and therefore a slight increase in the load may cause a new breakage in the fibre, whereas for smaller fragment lengths, the stress transfer is not efficient enough to rebuild the nominal stress in the fibre, and therefore saturation is reached and fragmentation stops.

In view of these results when the average fragment length is high, the splitting of a fragment can be achieved with a small increase of the applied load and on the contrary, after a critical fragment length is reached, a high amount of load is needed to produce new cracks in the fibre.
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The effect of fibre modulus and two size of fibre radius on the fibre axial and interphase shear stress distributions from BEM analysis are shown in Fig. 5. The interphase shear stress is singular at the free edge.

Fig. 6 shows that the interphase stress transfer is affected by the interphase modulus throw the ratio $E_i/E_m$. The Poisson’s ratio of the interphase is assumed to be the same as that of the matrix. It was found that the maximum fibre axial stress increase when the interphase/matrix stiffness ratio $(E_i/E_m)$ increases. But the increase of fibre axial stress is not monotonic with the increase of interphase stiffness. The fibre axial stress increases as $E_i/E_m$ increases from 1 to 25. However, in the range of $E_i/E_m$ =25-100 the fibre axial stress increase slowly and then remains nearly constant when $E_i/E_m >200$.

![Fig. 6. Maximum fibre axial stress](image)

The effect of interphase thickness on the fibre axial and interphase shear stress distributions for $E_i/E_m =1.56$ are shown in Fig. 7. The main effect of the interphase thickness is on the interphase/matrix shear stress. There is no influence of fibre crack on the interphase/matrix shear stress at bigger interphase thickness, i.e. the oscillate character of the interphase/matrix shear stress disappear.

It was shown in [4] that interphase debonding accompanies the fibre fracture. Debonding occurs at the fibre/matrix interface due to the shear stress singularity at the free edge. The effect of debond length $(a=10, 30, 50$ μm) on the fibre axial and interface shear stress distributions is shown in Fig. 8. If the debond length increases the oscillation of interface stresses decreases.
The strain energy release rate principle is used to study the mechanism of interfacial failure in fragmentation tests. In fact it can be used as an energy criterium for interfacial debonding.

Numerical solution of the energy release rate, computing using formula (3.3) is plotted versus the size of the crack in Fig. 9. In view of these results, energy release rate tends to infinity when crack size tends to a zero value, which means that small debonding cracks will form immediately after the fibre fracture, and, after that, the slope of the curve is decreasing steadily as the crack size increases, what implies a stable debond crack growth during the whole test.

5. Conclusions

The capability of BEM to obtain accurate solutions of the stress state in this problem by means of an axi-symmetric model has been clearly illustrated in this work, even for situation where complex singular stresses and contact are present.
As can be seen from the results shown, BEM solution of the stress state in the sample during the process of fragmentation and debond crack growth are very precise and permit an accurate evaluation of the crack extension criteria.

Therefore, the stress solutions computed with this BEM algorithm, will make it possible, in view of fibre and matrix elastic and fracture properties, to give quantitative values for interfacial strength, when a stable growth of an interface crack is obtained as a result of the test, or, at least, to obtain bounds for this property when another kind of failure takes place.

REFERENCES


